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Visual Simulation of Magnetic Fluids Using Dynamic Displacement Mapping for Spike Shapes

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<Summary> Although several simulation methods have been proposed to analyze the behavior of magnetic fluids in the computational physics field, it is still not reasonably tractable to simulate the spiking phenomenon using fully physically based methods. To synthesize spike shapes, a procedural approach has been proposed in the computer graphics field recently for visual simulation, which, however, cannot simulate dynamic arrangement and deformation of spikes since the spike shapes are determined beforehand. To overcome this drawback, we propose an improved model to incorporate the dynamic behavior of the spikes. The model is based on the assumption that the spikes are attracted by an external magnetic field and that each spike repels the others because of their magnetization. The position of the spikes on the liquid surface are determined in the simulation, and the spike shape is mapped according to the direction of the magnetic field lines. The coordinates at which the spike protrudes from the liquid surface are calculated in the fluid simulation.

Keywords: magnetic fluid, fluid simulation

1. Introduction

In the field of computer graphics, the visual simulation of fluid phenomena is an important research topic. Several methods have been proposed for representing realistic incompressible fluids by solving the Navier–Stokes equations. In some fluid phenomena, the interaction between a magnetic field and a fluid is important: for example the aurora borealis [1)] is an interplay between the fluid behavior of plasma and the magnetic field of the earth. In this paper, we focus on the interaction between magnetic fluids and the external magnetic field, and propose a method for representing the resulting fluid behavior. The difference between a magnetic fluid and the aurora is the presence and absence of a free boundary surface, respectively.

A magnetic fluid is a colloidal solution consisting of microparticles of ferromagnetic bodies, a surfactant that covers the magnetic microparticles, and a solvent that acts as the matrix (see **Fig. 1**). When a magnet is located near a magnetic fluid, the surface of the fluid forms spikes (such as horns) along the direction of the magnetic field generated by the magnet. This is known as the "spiking phenomenon" (see **Fig. 2**). This phenomenon is used to create artworks, where interesting shapes are generated by applying magnetic forces to the fluids.

Within the field of computer graphics (CG), Ishikawa et al. proposed a method for synthesizing these spike shapes by employing a procedural approach [2)]. Since the spike shapes were determined beforehand, this method could not simulate the dynamic arrangement and deformation of the spikes. In this paper, we propose a model to incorporate such dynamic arrangement and deformation of the spikes.

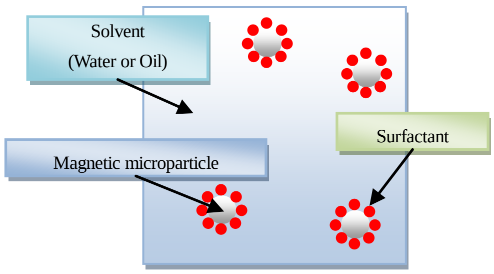


Fig. 1 Composition of magnetic fluid


Fig. 2 Spiking phenomenon of magnetic fluids (photograph)

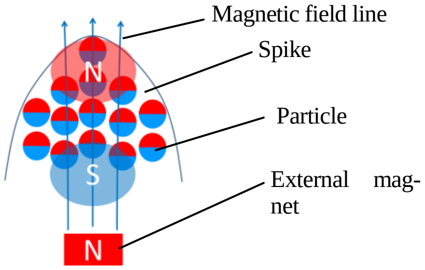


Fig. 3 Spike and magnetization of particles

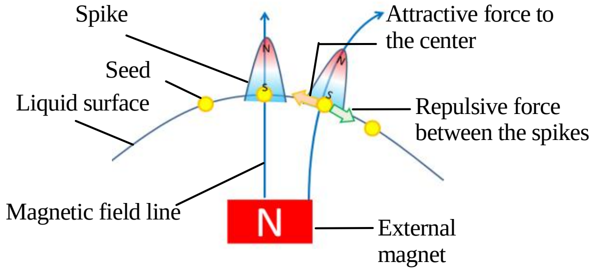


Fig. 4 Forces acting on “seed”

2. Related Work

Many methods have been proposed to simulate incompressible fluids such as smoke and flames [3), 4)]. Goktekin et al. proposed a simulation method for viscoelastic fluids by incorporating an elastic term into the Navier–Stokes equations [5)]. Stam and Fuime introduced the smoothed-particle hydrodynamics (SPH) method into the CG field for representing flames and smoke [6)]. Müller et al. proposed an SPH-based method to simulate fluids with free surfaces [7)]. A method for calculating the density distribution as voxel data is typically used for the fluid modeling of phenomena such as the aurora borealis. However, fluid phenomena such as water which have free boundary surface are often simulated using the particle method. Our method also uses an SPH-based method [2)] to simulate magnetic fluids.

In the field of physics, the characteristics of magnetic fluids have been studied since 1960. Han et al. modeled the formation of a chain shape between colloidal particles according to the magnetization of the particles [8)]. Combined with the lattice Boltzmann method, they showed that colloidal particles would form lines along the magnetic field. However, their method cannot accurately reproduce spike shapes. Yoshikawa et al. combined the moving-particle semi-implicit (MPS) method with the finite element method (FEM) and simulated magnetic fluids [9)]. Even when using 100,000 particles and a mesh with 250,000 tetrahedra, they were able to reproduce only a single spike.

We observed that the number of spikes increases when a strong magnetic force is applied to magnetic fluids, and the spikes extend along magnetic field lines. It seems these dynamic behaviors are not reasonably tractable using fully physically based method. Hence, we instead propose a physically plausible method to reproduce these dynamic features.

3. Our Model

Our method is based on the SPH method, which is used to calculate the magnetic forces acting on simulation particles (hereinafter SPH particles). Each particle represents a small magnetic fluid element including microparticles, surfactant and a solvent, and its motion is calculated by taking into account the properties of both fluid and magnetic body. After calculating the fluid surface using SPH particles, we map each spike shape to the liquid surface. Unlike the previous method [2)], we model each spike individually, hence allowing the spikes to rearrange dynamically. When using the SPH method, we place “markers” on the fluid surface for mapping the spike shapes. Each spike shape is represented as a superposition of trigonometric functions. We need to calculate only the center of the mapping (i.e., the place of the marker) which we denote as a “seed.”

By calculating the movement of the seeds, considering the fluid behavior and the force between seeds, we map the spike shapes to the seeds at each time step of the simulation. The SPH particles and seeds are simultaneously computed. The number of seeds considered for the calculations is proportional to the external magnetic force.

3.1. Movement of “seeds”

As a seed is a mapping point, it should be located on the liquid surface. In our method, a seed is modeled as a special type of SPH particle, whose mass is lower than that of a standard SPH particle. That is, mass of seed particles is αm if mass of standard SPH particle is m. Let α be 0.9 in this paper. At a seed location, the pressure from other SPH particles is calculated. However, the pressure from the seeds to the other SPH particles is not considered, since these seeds are not physically based entities for representing the fluid. Note that the seeds are considered in the density calculation. A bar magnet is placed outside the fluid; we call this magnet an external magnet, which is used to magnetize the fluid particles. The fluid particles are hence magnetized due to this external magnet. In each spike, the (real) magnetized micro particles are arranged along the magnetic field line and oriented by attractive forces (**Fig. 3**). Therefore, we assume that the spike itself forms a single magnet, thus explaining why separate spikes repel each other. In addition, attractive forces act toward the intersection point between the

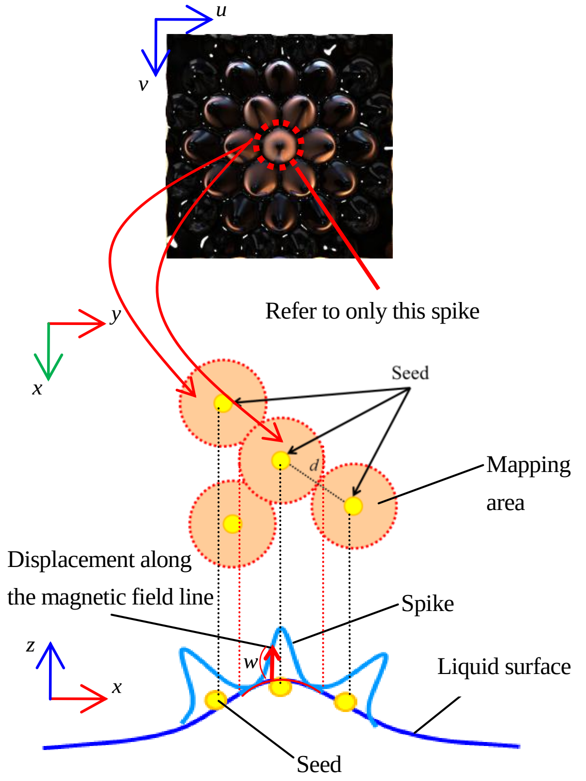


Fig. 5 Spike shape reproduced by a function (top), Conceptual diagram of mapping to the liquid surface (middle: top view, bottom: cross-sectional view)

liquid surface and the center of the magnetic field line induced by an external magnet, as the top of the external magnet is a well-balanced point (**Fig. 4**).

We propose that the stable point of the spike can be determined in relation to the following two forces: the attractive force toward the center and the repulsive force between the spikes. In this method, we calculate the mapping points (seeds) by calculating the interaction of the forces between the seeds, rather than those between the spikes. We consider the Lennard-Jones potential (Eq. (1)), where d is the distance between any two seeds, and p, q, ε, and σ are parameters. The attractive force and repulsive force are included in the Lennard-Jones potential.

$$U(d) = 4\varepsilon\left\{\left(\frac{\sigma}{d}\right)^{p} - \left(\frac{\sigma}{d}\right)^{q}\right\}. \tag{1}$$

The Lennard-Jones potential is typically used for computing the interaction between two atoms. The force F acting on the seed on account of this potential is calculated by the following formula.

$$F(d) = -\nabla U(d). \tag{2}$$

By calculating the force from the distance between the seeds and considering their movement due to an external force in the SPH method, we can determine the coordinates of the seed at which the spike is formed.

Table 1: Parameter setting used in equations

param.	meaning	value
dt	timestep	0.01 s
p	order of the attractive force term	2.0
q	order of the repulsive force term	4.0
ε	depth of the potential well	0.02 J
σ	collision diameter	2.6×10^{-3} m
β	proportional coefficient in Eq. (4)	15.0×10^{-3} m

3.2. Mapping the spike shapes

After calculating the coordinates of the seeds, we map the spike shape. The method is based on displacement mapping, where the fluid surface of the seed periphery is directly deformed. In addition, we allow the fluid surface to displace in the direction of the magnetic field line (because the spike of the magnetic fluids protrudes in this direction), as indicated by the height map (**Fig. 5** bottom). The surface shape for mapping is taken from an analysis of the case in which a vertical magnetic field is applied to the computation of horizontal surface of a magnetic fluid [10]. The function is known as the one that we can analytically seek for. This shape can be represented as a height field as follows:

$$w(u,v) = C_0\left(\cos\frac{k}{2}\left(\sqrt{3}u+v\right) + \cos\frac{k}{2}\left(\sqrt{3}u-v\right) + \cos kv\right). \tag{3}$$

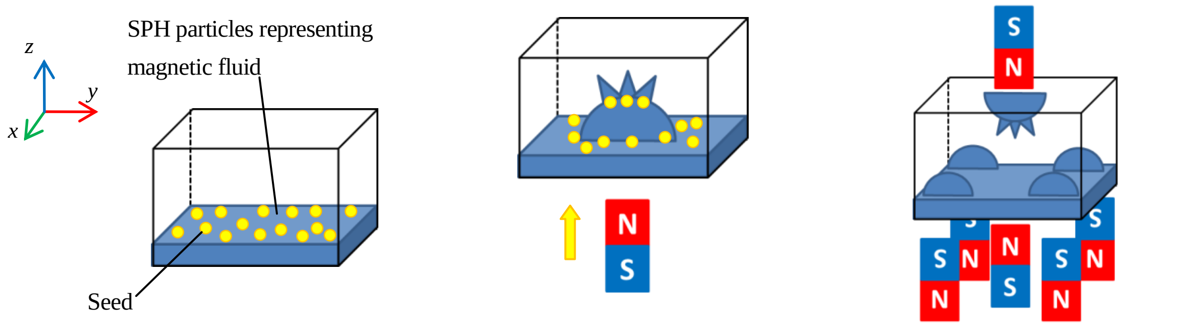
where u, v, and w are the three dimensions. C_0 indicates the amplitude of spike and k represents the frequency. We assume that C_0 is proportional to the magnitude of the magnetic field as follows,

$$C_0 = \beta\left|\mathbf{H}(\mathbf{r})\right|, \tag{4}$$

where β is the proportional coefficient, and $|\mathbf{H}(\mathbf{r})|$ is the magnitude of the magnetic field at seed position $\mathbf{r}$. $\mathbf{H}(\mathbf{r})$ is determined by the position of an external magnet and the magnetization of each SPH particle. Please refer to the Appendix for the calculation of this magnetic vector. We map each of the spikes by calculating Equation (3), referring to the height map within a certain radius of the center of the top of the spike (Fig. 5 top). A radius that contains a single spike has length equal to half of the cycle length in Equation (3). We create a spike shape by selecting the shortest distance between the seeds as the wavelength. When the shortest distance is d_{min}, the parameter k of the frequency in Eq. (3) is determined as follows,

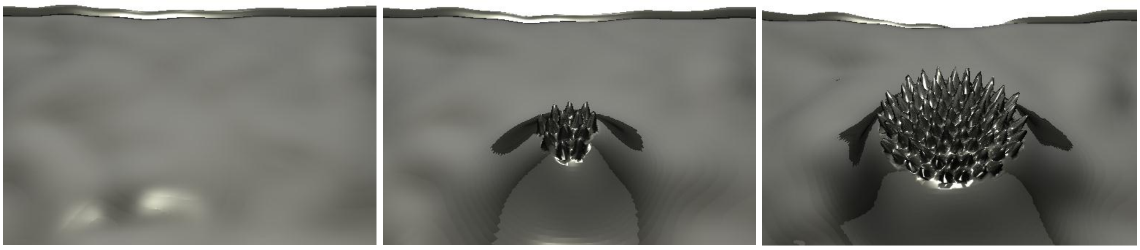
$$k = \frac{4\sqrt{3}\pi}{3d_{\min}}. \tag{5}$$

At each seed point, we map the spike shape to the seed position corresponding to the origin of the height map (Fig. 5 middle).



(a) Initial state (b) Magnet placement for Figure 7 (c) Magnets placement for Figure 9

Fig. 6 The simulation space for the proposed method, (a) The magnetic fluids, which are represented by a set of SPH particles, are stored in a cubic container, (b) The magnet is placed underneath the container, (c) The magnets are placed in a setup so as to recreate the artwork in Fig. 9



(a) 300 timestep (b) 330 timestep (c) 360 timestep

Fig. 7 Resulting images from the proposed method, showing simulation results when the magnet approaches from the bottom of the magnetic fluid

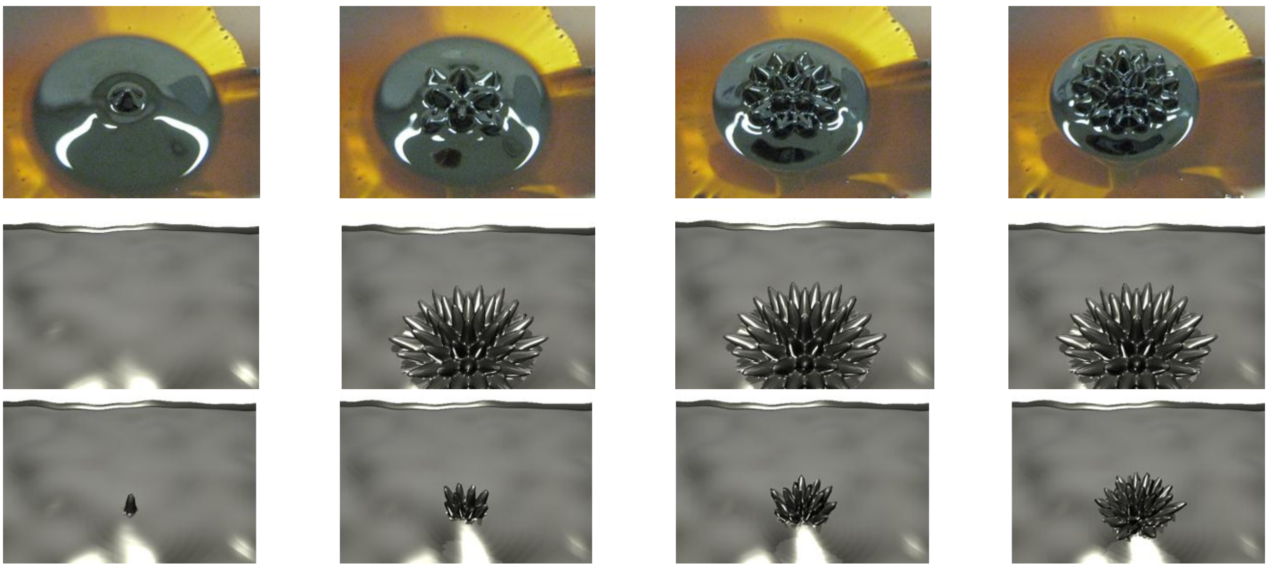
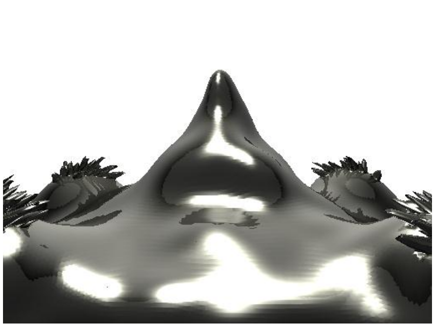


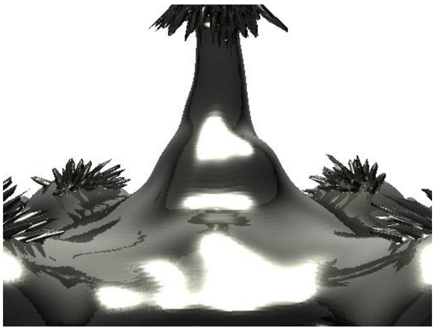
Fig. 8 Comparison of real spike phenomena (top), previous method (middle) and the proposed method (bottom)

4. Results

For the simulation, we used C++ and OpenGL for CPU programming, a CUDA-based GPU for the SPH method and the calculation of the motion of the seeds and SPH particles, and POV-Ray for rendering. The average computation time of the simulation for a single time step was 0.16 s when using a standard PC (CPU: Intel® Core 2 Duo™ 3.33GHz, RAM: 3.25GB, GPU: NVIDIA GeForce GTX 480). The surfaces of the magnetic fluids were extracted using the method proposed by Yu et al. [11]. Using the above mentioned PC, the average computation time for the surface construction of each frame was 2 min. The parameters used in our simulation are shown in **Table 1**.

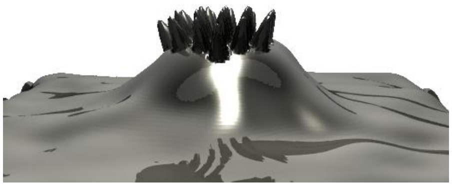


(a) 350 timestep

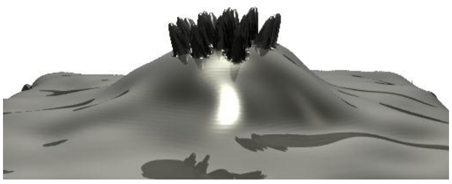


(b) 390 timestep

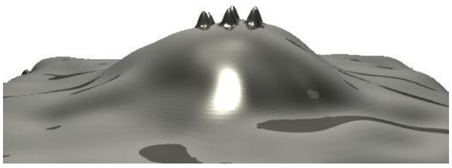
Fig. 9 Simulation results emulating an artwork, we set one magnet above and four magnets below the magnetic fluid



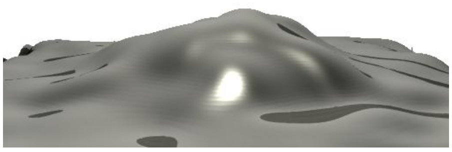
(a) 370 timestep



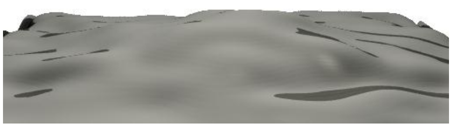
(b) 390 timestep



(c) 410 timestep



(d) 430 timestep



(e) 450 timestep

Fig 10 Simulated images showing the case where the magnet is moved away from the bottom of the magnetic fluid and the spikes disappear

The initial fluid surface is shown in **Fig. 6** (a). The magnetic fluids, which are represented by a set of SPH particles including the seeds, are stored in a cubic container. To achieve simulation results such as those shown in **Figs. 7** and **8**, the magnet is placed underneath the container. To emulate the art work in **Fig. 9**, we set the magnets as shown in Fig. 6 (c).

Fig. 8 shows the comparisons between real magnetic fluids, the previous method and proposed method by changing the magnetic force. The images in the middle row of images in Fig. 8 show the result using the previous method [2)]. The previous method has a restriction in which the number of spikes is fixed, hence cannot represent varying number of spikes. In addition, the arrangement of the spikes cannot be dynamically changed in this method. We can reproduce that the number of spikes increases when a strong magnetic force is applied to the magnetic fluids, as well as that the dynamic arrangement of spikes.

Fig. 9 shows the results to emulate an art work [12), 13)].

Fig. 10 shows simulation result when the magnet goes away from the bottom of the magnetic fluid. The proposed method can achieve a wide range of expression since our method can change the loca-

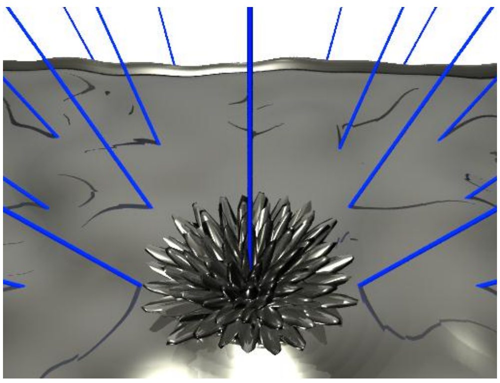


Fig. 11 The strokes show the magnetic field lines, we can see that the spikes extend along the magnetic field lines

tion of spikes dynamically.

Fig. 11 shows the result images of spike shape with magnetic field lines. We can see that each spike extends along the magnetic field lines, like actual magnetic fluids.

5. Conclusions and Future Work

By combining the SPH method with moving the mapping point of spikes, we have represented the dynamic deformation of spikes in magnetic fluids. These spikes were successfully incorporated by incorporating the calculation of the magnetic field into the SPH method. By moving mapping points along the fluid flow, we calculated the coordinates for mapping the spike shape.

In the future study, we would like to include other factors that depend on the magnetic field, such as surface tension and magnetic susceptibility. Moreover, we are planning to make interactive applications for studying magnetic fluids.

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Appendix

A. SPH method considering the magnetic force

The behavior of incompressible fluids is described by the following equations.

$$\nabla \cdot \mathbf{u} = 0, \tag{A1}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}. \tag{A2}$$

Eq. (A1) is the continuity equation and Eq. (A2) (Navier-Stokes equation) describes the conservation of momentum. $\mathbf{u}$ is the velocity vector, t is time, ρ is the fluid density, p is the pressure and ν is the kinematic viscosity coefficient. $\mathbf{F}$ is the external force that includes the gravity, the magnetic force, and the surface tension. We solve these equations by using SPH method.

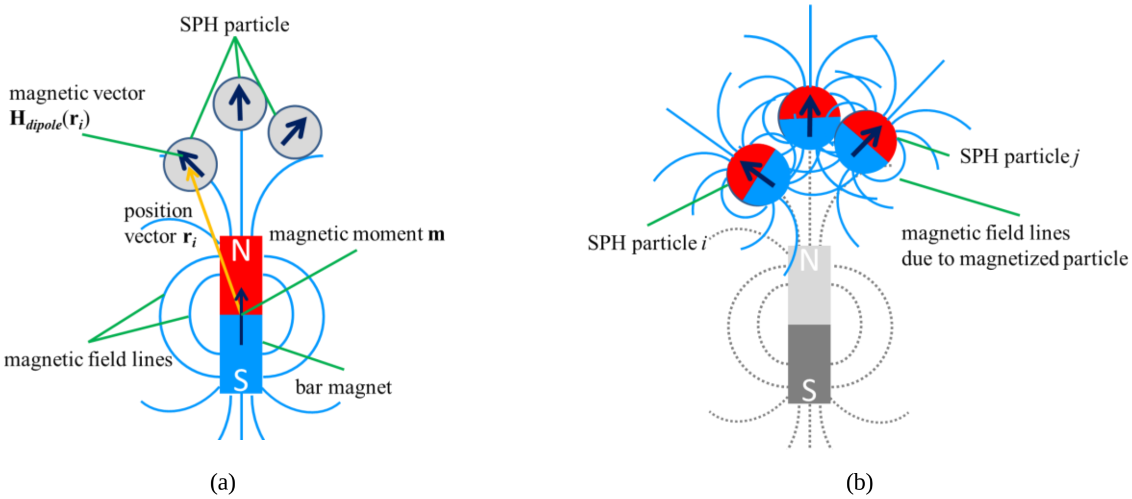

Fig. A Calculation of the magnetization and the magnetic force, (a) First, we calculate the magnetic field vector at each SPH particle induced by a magnetic dipole, (b) Next, we calculate the influence of other particles from the magnetized particles

Each SPH particle represents a small magnetic fluid element, and motions of the SPH particle and the mapping point “seed” are calculated by taking into account the magnetic force. By applying an external magnet, SPH particles are magnetized and work as small spherical magnet. Therefore, our method first computes the magnetic field at equilibrium state, taking into account the magnetization of SPH particles. After that, the magnetic force for each SPH particle is calculated. We call the magnetic vector induced by the magnetic bar as a background magnetic vector. Let us assume that the origin is at the midpoint between the north and the south poles. Then, the background magnetic vector $\mathbf{H}_{dipole}(\mathbf{r})$ at position $\mathbf{r}$ is expressed by the following equation.

$$\mathbf{H}_{dipole}\left(\mathbf{r}\right) = -\frac{1}{4\pi\mu}\nabla\frac{\mathbf{m}\cdot\mathbf{r}}{r^3}, \tag{A3}$$

where μ is the permeability of the magnetic fluid, $\mathbf{m}$ is the magnetic moment and $r = |\mathbf{r}|$. Each SPH particle is magnetized due to the background magnetic vector field and induces an additional magnetic vector field. Thus, in order to obtain the final magnetic vector $\mathbf{H}(\mathbf{r}_j)$ at particle j, the magnetic interactions between particles have to be computed by solving the following equation,

$$\mathbf{H}\left(\mathbf{r}_j\right) = \mathbf{H}_{dipole}\left(\mathbf{r}_j\right) - \frac{V}{4\pi\mu}\sum_{\substack{i=1 \\ i\neq j}}^{N}\nabla\frac{\chi\mathbf{H}\left(\mathbf{r}_j\right)\cdot\mathbf{r}_{ij}}{r_{ij}^{3}}, \tag{A4}$$

where V is the volume of a particle and we assume that the volume of all particles are equal, $\mathbf{r}_i$ is the position of particle i, N is the total number of particles, χ is the magnetic susceptibility. $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$, and $r_{ij} = |\ \mathbf{r}_j - \mathbf{r}_i\ |$. We calculate Eq. (A4) by taking into account the influences from all the particles. We use the Gauss-Seidel method to solve Eq. (A4).

Finally, we obtain the magnetic force based on the distribution of the magnetic field by the following equation [10],

$$\mathbf{F}_{mag}\left(\mathbf{r}_i\right) = -\nabla\frac{\mu\left|\mathbf{H}\left(\mathbf{r}_i\right)\right|^2}{2}. \tag{A5}$$

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