Animating Impossible Figure from Twisted Torus to Penrose Stairs

Kana Nakatsu [*]
Tokyo Denki University

Tokiichiro Takahashi [†]
Tokyo Denki University
UEI Research

Tomoaki Moriya [‡]
Tokyo Denki University

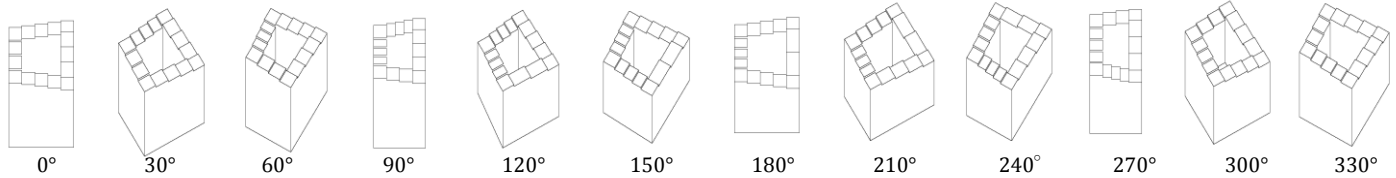| 0° | 30° | 60° | 90° | 120° | 150° | 180° | 210° | 240° | 270° | 300° | 330° |

Figure 3. Rotation of Penrose Stairs

1. Introduction

When a start prism and an end prism's vertices are in the same values on the projection screen, the shape model is viewed as an impossible figure. Therefore, if we can create the optical illusion object according to an arbitrary viewpoint, it is possible to create impossible figure that is viewed from the arbitrary viewpoint. We have already proposed and developed a method which animates impossible rectangle twisted torus figure from any viewpoint interactively and in real time[Nakatsu,et al. 2012]. Although the method of creating an optical illusion based on the optical illusion objects is a simple and classic method, this is powerful and robust all the more. Even when a viewpoint is moved to a wide range of area, the animation of an impossible figure can be created. We have extended the application of this principle to Penrose Stairs, and this paper presents that it was successfully realized when the viewpoint was moved freely.

2. 3D Shape Model for Optical Illusion Object

2.1 3D Shape Model

Three dimensional shape model as the optical illusion model to be used in our method arranges 4 or more prisms parallel to X, Y, and Z-axis. There, each prism is connected vertically.

Two prisms connected with each other are fundamentally called 'start prism' and 'end prism' respectively. The start prism, which is the start of the object, is arranged along Y-axis, and the end prism, which is the end of the object, is arranged along X-axis. There is a gap between the start prism and the end prism, and they are not connected (Figure 1(b)). The viewpoint where the start prism and the end prism look connected is called an optical illusion viewpoint. Here, the optical illusion can be seen as an impossible rectangle twisted torus figure (Figure 1(a))

We call the end prism and the prism connected to it as 'flexible quadrangular prism' ('flexible prism' in short). We transform the shape model into the optical illusion object by stretching the flexible prisms according to the given viewpoint.
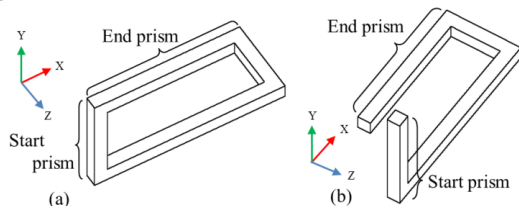

Figure 1. An Optical Illusion Object for An Impossible Rectangle Twisted Torus Figure.

2.2 Illusion Viewpoint and Projection

The sphere containing an optical illusion object is called 'bounding sphere'. The illusion viewpoint moves on the upper half of the bounding sphere except the pole. The illusion viewpoint looks at the center of the sphere all the time. The illusion viewpoint is given by a latitude ϕ, and longitude θ on the bounding hemi-sphere. The illusion viewpoint is expressed as (ϕ, θ), where $0 \leq \phi < \frac{\pi}{2}, 0 \leq \theta < 2\pi$. Note that the projection method is parallel projection.

3. Penrose Stairs

Penrose Stairs (Figure 2(a)) is a staircase shape impossible figure that has a property to continue ascending and descending. This figure is a more complicated shaped impossible figure that is derived impossible rectangle twisted torus figure (Figure 2(b)). We can model an optical illusion object of Penrose Stairs (Figure 2 (c)) based on our method [1]. Thus, a rotating animation of Penrose Stairs can be created.

We have made a rotating animation of impossible figure of Penrose Stairs that maintained the nature of impossible figure at all times, which was made possible by applying our proposed method to the optical illusion objects as its components. Figure 3 shows the application result at each $\theta = 30°$. We used the post effect for an ideal line drawing result. We have also created an interactive animation system for an impossible figure of Penrose Stairs. See the attached movies.

However, our proposed method as it stands is unsuitable for Penrose Stairs figure because its optical illusion object has a depth gap that makes distortion when the range of θ is $\frac{\pi}{2} \leq \phi < \pi$ (the part shown with slanting lines in Figure 2 (c)). To add to our proposed method, we need to devise the way to show the bottom of the prism being in contact with the ground in a line, such as moving the bottom position to Y- axis direction.
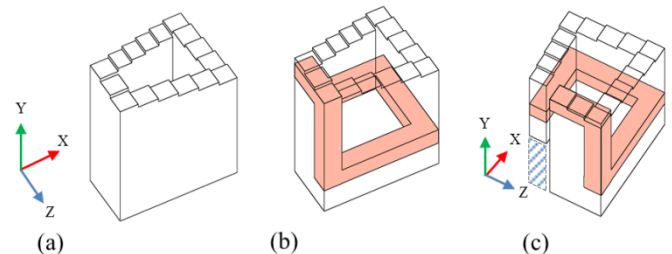

Figure 2. An Optical Illusion Object of Penrose Stairs Based on Impossible Rectangle Twisted Torus.

4. Conclusion

We have applied the method of creating an optical illusion based on the optical illusion objects to Penrose Stairs, and realized the interactive rotating animation of impossible figure. We have shown that our proposed method can be generally used to the impossible figures derived from impossible rectangle twisted torus figures. This method can also be applied to stereoscopic vision.

References

K. Nakatsu, T. Takahashi, T. Moriya, "A stereoscopic representation of impossible rectangle twisted torus figure", ACM SIGGRAPH 2012 Posters, Article 2 (2012).