A Fast Display Method of Sky Color Using Basis Functions

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Abstract

Computer graphics recently have began being used for visual environmental assessment or architectural designs. Displaying the sky as a background is indispensable in generating photorealistic images for such applications. In this paper, we propose a fast display method of the sky color by expressing the intensity distribution of the sky using basis functions, even if the sun position and/or the camera position are altered. In the proposed method, cosine functions are used as basis functions. The sun altitude is altered at certain intervals and the distributions of the sky color for each sun altitude is precalculated and stored efficiently using basis functions. The color of the sky in the view direction of an arbitrary sun position can be obtained from the stored distributions and displayed quickly.

1 Introduction

Computer graphics have become indispensable tools for visual environmental assessment and architectural designs. With computer graphics, it is easy to forecast a scene from an arbitrary viewpoint at different times of the day. For generating photorealistic images of natural scenes or buildings, the background sky is very important. Especially for architectural design, displaying the sky has become critical as many buildings are designed so that the sky or a surrounding scene is reflected in the building windows. The color of the sky depends on the altitude of the sun, the atmospheric conditions and the viewing direction, changes which results in the variation of the appearance of buildings that reflect the sky color.

In computer graphics, two simple methods have often been used to display the sky color. One involves simply painting the blue background, and the other specifies two colors at the top and bottom of the screen (for example, blue at the top and white at the bottom) and displays the sky by the linear interpolation of the two colors. In reality, however, the sky shows more subtle color variations. Especially in the evening, the setting sun can turn the color of the sky near the horizon a beautiful orange with its zenith becoming dark blue. This phenomenon can be very attractive. To display such subtle color variations, several methods of calculating sky color have been proposed [Klassen 87, Inakage 89, Kaneda 91, Tadamura 93]. A method to calculate not only the color of the sky viewed from the ground but also the color of the atmosphere viewed from outer space has also been proposed [Nishita 93]. The calculation of sky color, however, is time consuming as numerical integration is required to obtain the scattered light of the sun due to particles in the atmosphere. In order to make, therefore, an animation that incorporates a changing sun position and viewpoint, a great deal of time must be spent on the calculation of the sky color.

This paper proposes a new method of displaying the sky color. In the proposed method, the intensity distribution is precalculated for every sun altitude sampled at a certain interval. Each distribution is expanded into a Fourier series and is then stored efficiently, making it possible to store the distribution for each sun altitude using only small amount of memory. Furthermore, for a given field of view, the method can display the sky color quickly with an arbitrary sun position by applying the stored intensity distribution of the sky.

In section 2, previous methods of displaying the sky and their problems are mentioned. In section 3, the basic idea of the proposed method is described. Next, in section 4, a new method to represent the intensity distribution of the sky is proposed. In section 5, an optimal method of representing the intensity distribution of the sky using Fourier series and a method of calculating sky color are proposed. Finally, in section 6, the proposed method is applied to the display of natural scenes and buildings to demonstrate the usefulness of the method.

2 Previous Work

Klassen tried to display sky color by taking into account spectral distribution due to particles in the atmosphere [Klassen 87]. The method, however, has the following problems: the atmosphere is approximated as multiple layers of plane-parallel atmosphere with uniform density. Thus, the method is different from the actual physical phenomenon. An improved method of calculating sky color, which approximates the actual physical

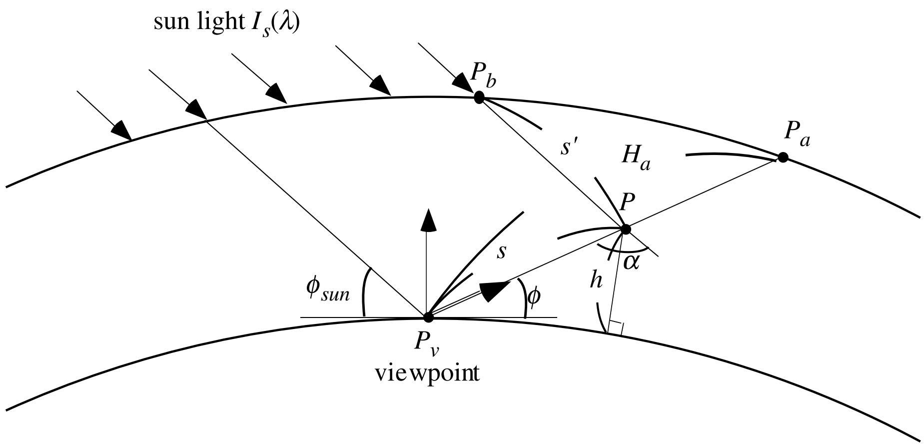

Figure 1: Geometry of the atmosphere.

phenomena, was proposed [Kaneda 91]. In the improved method, the atmosphere is considered a spherical-shell and the density distributions of both air molecules and aerosols vary exponentially with altitude. Tadamura et al. [Tadamura 93] discussed the relationship between the model proposed by Kaneda et al. [Kaneda 91] and the CIE standard [CIE 73]. They proposed physical parameters suitable for the CIE standard. These three methods can display a realistic sky color, but it takes a great deal of time since the numerical integration method is necessary.

3 Basic Idea

3.1 *Calculation of Sky Color*

The atmosphere consists of both air molecules and aerosols. The scattering due to the former obeys the Rayleigh scattering theory and the latter obeys the Mie scattering theory. As shown in Fig. 1, for Rayleigh scattering, the intensity of the sky, $L_\lambda(\mathbf{v})$, in the viewing direction, $\mathbf{v}$, is calculated by the following equation [Kaneda 91].

$$L_\lambda(\mathbf{v}) = \frac{I_s(\lambda)\, k\, F(\alpha)}{\lambda^4} \int_0^{H_a} \rho(s) \exp\left(- t(s',\lambda) - t(s,\lambda)\right) ds, \tag{1}$$

where

λ : wave length

$I_s(\lambda)$: intensity of the sunlight out of the atmosphere

k : constant for standard atmosphere

$F(\alpha)$: phase function of air molecules

H_a : distance between the top of the atmosphere, P_a , and viewpoint, P_v

ρ : density ratio of air molecules

s' : distance between a certain point in the atmosphere, P, and P_b

s : distance between P and P_v

Since the density ratio of air molecules varies exponentially with the height from the ground, the optical length, $t(s, \lambda)$, is calculated by the following equation.

$$t(s,\lambda) = \frac{4\pi k}{\lambda^4}\int_0^s \rho(l)\, dl = \frac{4\pi k}{\lambda^4}\int_0^s \exp\left(-\frac{h(l)}{H_0}\right) dl, \tag{2}$$

where H_0 is a constant called the scale height of the atmosphere and $h(l)$ is the height from the ground. Although both air molecules and aerosols must be taken into account in the actual physical phenomenon, only the scattering of air molecules is described here. For more details, see [Kaneda 91]. To calculate the color of the sky from Eqs. (1) and (2), the intensity of the light arriving at the viewpoint, which is the sun light scattered and attenuated due to particles, must be integrated along the viewing ray. As shown in Fig. 1, the sunlight travels along the path, P_bP, scatters at the point, P, toward the viewpoint, and finally arrives at the viewpoint after traveling along the path, PP_v. Thus, to calculate the intensity of the scattered light of the sun arriving at the viewpoint, P_v, the optical length of the path, P_bPP_v (see Fig. 1), must be calculated. The optical length is calculated by using Eq. (2). As mentioned previously, the density ratio of the particles in the atmosphere varies exponentially with the altitude, which results in difficulties in obtaining the analytical solution for the integral. By generating sample points on the paths along the viewing ray and the sun direction, therefore, the optical length must be calculated by the numerical integration method. In order to calculate the sky color precisely, a large number of sample points is required, which results an increase of the computation time.

3.2 *Proposed Idea*

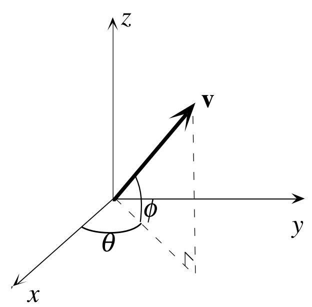

Figure 2: Coordinate system.

Since it takes time to calculate the color of the sky by using numerical integration every time the sky is displayed, the intensity distribution of the sky is calculated and stored in advance. The sky is considered to be a hemisphere with a large radius, so the viewpoint can be treated as the effective center of this hemisphere. Assuming a coordinate system where the viewpoint is at the origin, the view direction, **v**, is expressed with angle θ from x axis (azimuth angle) and angle ϕ from xy plane (elevation angle) as shown in Fig. 2. The intensity distribution of the sky is expressed with $L_\lambda(\mathbf{v};\, \phi_{sun})$ where ϕ_{sun} is the altitude of the sun, and λ is the wave length. As shown in Fig. 3, the intensity distribution of the sky can be considered as a weighted

$$L_\lambda(\theta,\phi_0;\,\phi_{sun}) = w^{(\lambda)}{}_0(\phi_{sun})\,\psi_0(\theta,\phi_0) + w^{(\lambda)}{}_1(\phi_{sun})\psi_1(\theta,\phi_0) + w^{(\lambda)}{}_2(\phi_{sun})\psi_2(\theta,\phi_0) + \cdots$$

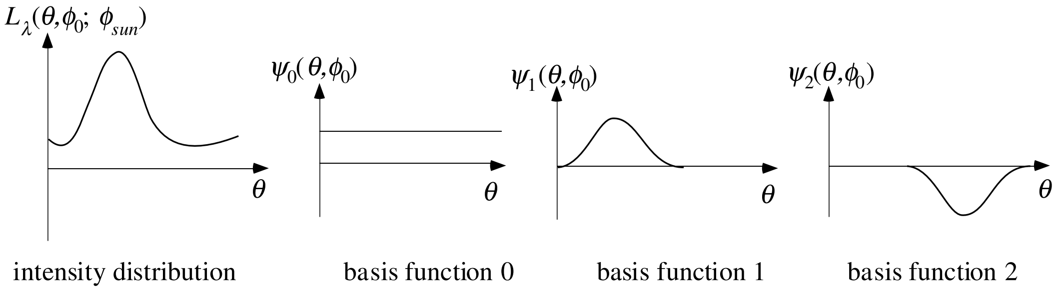

Figure 3: The intensity distribution of the sky and its basis functions.

sum of several basis functions. Note that Fig. 3 shows the distribution as a function of θ with a fixed ϕ_0. If it is possible to calculate the weights as a function of the altitude of the sun, the intensity distribution of the sky can be obtained quickly by summing the basis functions with weights, even if the position of the sun is altered. That is,

$$L_\lambda(\mathbf{v};\,\phi_{sun}) = \sum_{l=0}^{N} w_l^{(\lambda)}(\phi_{sun})\psi_l(\mathbf{v}), \tag{3}$$

where $\psi_l(\mathbf{v})$ is a basis function, $w^{(\lambda)}{}_l(\phi_{sun})$ is a weight function for the basis function and N is the number of basis functions required to express the intensity distribution of the sky. Once the weight, $w^{(\lambda)}{}_l(\phi_{sun})$, for each altitude of the sun is calculated and stored, the intensity distribution of the sky for an arbitrary position of the sun can be obtained by using Eq. (3). To reduce the cost of both the computation and the memory for the weights, the following two conditions must be satisfied:

- The intensity distribution of the sky should be expressed with the smallest possible number of basis functions.
- Any of the intensity distributions of the sky should be expressed with a linear combination of basis functions.

Nimeroff et al. [Nimeroff 94] proposed an efficient method of calculating the illuminance on surfaces due to light from the sky when the position of the sun is altered. In the method, the terms depending on the sun position are extracted from the distribution of the skylight and expanded into a series of basis functions. This method can express intensity distribution with ten basis functions. The method, however, doesn't take into account the spectral distribution of the skylight because it uses the CIE standard distribution as the intensity distribution of the sky. Furthermore, to extract the terms depending on the position of the sun, the intensity distribution must be expressed with an analytical expression, such as the CIE standard distribution. In general, since the distribution of the skylight is calculated by numerical integration, it is difficult to extract the terms depending on the position of the sun analytically.

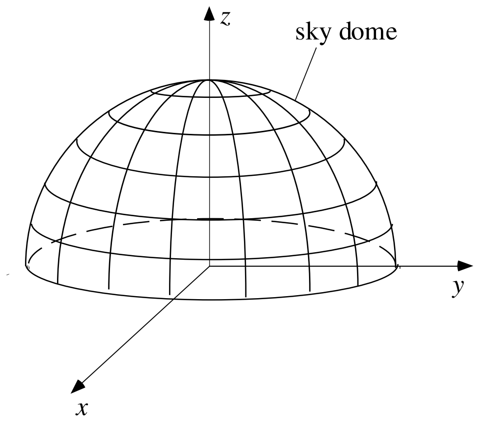

Figure 4: The sky dome divided into grids.

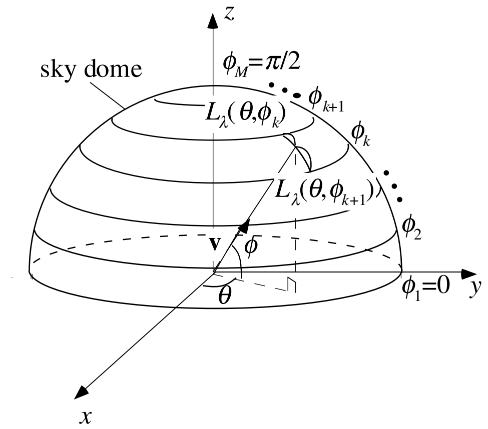

Figure 5: The sky dome divided in the direction of ϕ.

In the proposed method, the intensity distribution of the sky is expressed by its expansion into a Fourier series. Any intensity distribution can be expressed by the Fourier series. The costs of both the calculation and the memory for the weights greatly depend on the types of the basis functions. In the next section, the basis functions for expressing the intensity distribution of the sky are discussed.

4 Representation of Intensity Distribution of Sky Using Basis Functions

Let the sun direction be $(\theta_{sun}, \phi_{sun})$ and suppose it is always on the xz plane (i.e., $\theta_{sun}=0$). And let the intensity distribution of the sky be $L_\lambda(\theta, \phi; \phi_{sun})$. When θ_{sun} is not zero, the intensity distribution of the sky can be obtained by $L_\lambda(\theta-\theta_{sun}, \phi; \phi_{sun})$. As mentioned previously, the intensity distribution of the sky is precalculated for the altitude of the sun at a certain interval and is stored using basis functions. The following are three ways to express the intensity distribution of the sky using basis functions.

1. As shown in Fig. 4, the hemispherical dome of the sky is divided into grids and the color of the sky at each grid point is precalculated. The intensity distribution of the sky is expressed by the colors at those grid points. The sky color in an arbitrary direction is obtained by the linear interpolation from the colors at grid points. This method is equivalent to using delta functions as basis functions.
2. Using spherical harmonic functions, $Y_{lm}(\theta, \phi)$, as basis functions, the intensity distribution of the sky is expressed by the following equation.

$$L_\lambda(\theta,\phi;\phi_{sun}) = \sum_{l=0}^{N} \sum_{m=-l}^{l} w_{lm}^{(\lambda)}(\phi_{sun}) Y_{lm}(\theta,\phi). \tag{4}$$

Weight for each spherical harmonic functions is precalculated.

3. As shown in Fig. 5, the sky dome is divided into slices only in the ϕ direction. In the θ direction, the intensity distribution of the sky is expressed by cosine functions. That is,

$$L_\lambda(\theta,\phi_j;\ \phi_{sun}) = \sum_{i=0}^{N_j} c_i w_{ij}^{(\lambda)}(\phi_{sun}) \cos(i\theta), \qquad (j = 1, ..., M), \tag{5}$$

$$c_i = \begin{cases} \sqrt{\dfrac{1}{\pi}}\ , & i = 0 \\ \sqrt{\dfrac{2}{\pi}}\ , & i = 1, 2, ..., N_j \end{cases} \tag{6}$$

where M is the number of divisions in the direction of ϕ, N_j the number of cosine functions required to represent the intensity distribution of the sky in slice, j, and c_i is the normalizing factor. Weight for each cosine function is precalculated.

The first method is the simplest and has the advantage of being able to calculate the intensity distribution of the sky quickly, as the sky color in an arbitrary direction can be obtained just by linear interpolation from the stored color of the sky at each grid. The disadvantage of the method, however, is the large memory requirement for storing sky colors when the number of grid points is increased to guarantee high accuracy. The second method has often been used in computer graphics. Kajiya et al. used spherical harmonic functions to represent the phase function of particles in clouds [Kajiya 84]. Both Cabral et al. and Sillion et al. used spherical harmonic functions to represent the reflectance functions [Cabral 87, Sillion 91]. But the second method's disadvantage is that high degrees of spherical harmonic functions are required to express the intensity distribution of the sky due to a sharp peak of intensity around the sun. Furthermore, spherical harmonic functions are complicated expressions, resulting in an increase of the calculation time to obtain the distribution using Eq. (4). In our experiment using the first method, described in section 5.3, approximately 17[MB] of memory is required for the sky colors at grid points for all altitudes of the sun. Using the second method, it takes more than one minute to obtain intensity distribution using Eq. (4), while other two method take only a few seconds. We propose a third method for expressing the intensity distribution of the sky that contains advantages of both the first and the second methods. As an alternative of the third method, it is possible to divide the sky dome in the θ direction instead of the ϕ direction. In this case, the intensity distribution in each divided area is expressed by a Fourier series. The sky color calculated by each Fourier series must coincide at the zenith as the zenith is shared with all divided areas. Such a point can be avoided by subdividing in the ϕ direction. Another alternative is to divide in the both θ and ϕ directions. The intensity distribution of the sky is then expanded into a Fourier series as for ϕ_{sun}. Our experiment, however, requires a larger number of cosine functions to guarantee high accuracy, resulting in an increase of computation cost. The reason cosine functions is selected is that, as described in section 5, it is easy to evaluate the approximation error. Furthermore, due to the asymmetrical property of the intensity distribution of the sky, the sky color in the θ ($\pi < \theta < 2\pi$) direction can be obtained by $L_\lambda(2\pi\text{-}\theta, \phi)$, so only half of the sky in θ ($0 \le \theta \le \pi$) is taken into account. From the property of the Fourier series, the continuity of the intensity distribution of the sky at $\theta = 0$ and $\theta = \pi$ are guaranteed.

Next, the calculation method of the weights is described. From the property of orthogonal functions, weights $w^{(\lambda)}{}_{ij}(\phi_{sun})$, for the cosine function $\cos(i\theta)$, are obtained by the following equation [Onodera 88].

$$w_{ij}^{(\lambda)}(\phi_{sun}) = c_i \int_0^{\pi} \cos(i\theta)\, L_\lambda(\theta, \phi_j; \phi_{sun})\, d\theta, \quad (i = 0, ..., N_j;\ j = 1, ..., M). \tag{7}$$

From the above equation, weights are calculated by using numerical integration. That is,

$$w_{ij}^{(\lambda)}(\phi_{sun}) = c_i \Delta\theta_w \sum_{k=0}^{K_w} \cos(ik\Delta\theta_w)\, L_\lambda(k\Delta\theta_w, \phi_j; \phi_{sun}), \tag{8}$$

where K_w is the number of sample points for numerical integral calculation and $\Delta\theta_w = \pi/K_w$. The calculation time of weights can be reduced by using a table as follows.
Let us define p_{ik} as the following equation.

$$p_{ik} = c_i\, \Delta\theta_w \cos(ik\Delta\theta_w), \qquad (i = 0, ..., N_j;\ k = 0, ..., K_w). \tag{9}$$

Substituting Eq. (9) for Eq. (8),

$$w_{ij}^{(\lambda)}(\phi_{sun}) = \sum_{k=0}^{K_w} L_\lambda(k\Delta\theta_w, \phi_j; \phi_{sun})\, p_{ik}, \tag{10}$$

p_{ik} can be precalculated and are stored in the form of tables. This reduces the calculation time for weights as cosine functions do not need to be evaluated in every weight calculation.

5 Determination of The Optimal Number of Cosine Functions

Even if the intensity distribution is expressed by Fourier series, the computation time and memory requirement for weights of each cosine function could increase when the number of cosine functions become larger. In addition, the computation time for obtaining the sky color in the viewing direction by using Eq. (5) could also increase. To address this problem to a specified accuracy, we propose an optimal method for determining the minimum number of cosine functions, N_j (j=1, ..., M) (see Eq. (5)).

5.1 *Algorithm*

In general, the intensity distribution of the sky has a sharp peak of intensity around the sun and a lenient distribution elsewhere. It is therefore expected that $L_\lambda(\theta, \phi_j; \phi_{sun})$ in Eq.

(5) around the sun requires higher frequency of cosine functions, while lower frequency of cosine functions are sufficient for other areas. Furthermore, the higher the altitude of the sun, the smaller the intensity distribution changes in the direction of θ. For example, when the altitude of the sun is 90 degrees, the intensity distribution changes only in the ϕ direction. As a result, $L_\lambda(\theta, \phi_j; \phi_{sun})$ can be expressed by a cosntant term corresponding to $\cos(0\theta)$. By making use of this property of the intensity distribution of the sky, the numbers of cosine functions, N_j, are determined by the following procedure.

Execute the following steps for each $j = 1, ..., M$;

step 1: $i=0$

step 2: Calculate $w^{(\lambda)}_{ij}$ by using Eq. (10).

step 3: Calculate the error $E^{(\lambda)}_{ij}$ of the intensity distribution of the sky expressed by cosine functions up to i (see section 5.2).

step 4: If $E^{(\lambda)}_{ij}$ is less than a specified threshold ε, then $N_j=i$. Otherwise, go to step 2 after $i=i+1$.

When a weight with index $i+1$ is calculated in step 2, it is not necessary to re-calculate all the weights up to index i. That is, the weights up to index i are available even if $\cos(i+1)\theta$ is added. The above procedure is terminated when the specified accuracy is satisfied. By using this method, the number of cosine functions required are adaptively determined. In other words, areas where the distribution changes drastically are expressed by higher frequency of cosine functions while the other areas are expressed by lower frequency of cosine functions, making it possible to express the intensity distribution of the sky precisely.

5.2 *Error Evaluation*

The method of evaluating the error, $E^{(\lambda)}_{ij}$, used in step 3 in the previous section is described here.

The error is evaluated by using the mean square error between L obtained by Eq. (1) and the intensity approximated by the proposed method. When the intensity distribution of the sky is expressed by Fourier series up to i, the error, $E^{(\lambda)}_{ij}$, is expressed by the following equation.

$$E^{(\lambda)}_{ij} = \int_0^{\pi} \left| L(\theta,\phi_j;\, \phi_{sun}) - \sum_{k=0}^{i} c_k w_{kj} \cos\,(k\theta) \right|^2 d\theta. \tag{11}$$

Since cosine functions are orthogonal to each other and normalized by the factor, c_i, Eq. (11) is rewritten as follows [Onodera 88].

$$E^{(\lambda)}_{ij} = \int_0^{\pi} \left| L(\theta,\phi_j;\, \phi_{sun}) \right|^2 d\theta - \sum_{k=0}^{i} w_{kj}^2. \tag{12}$$

The error , $E^{(\lambda)}{}_{ij}$, is calculated directly from Eq. (12). Note that $\int_0^{\pi} \left| L(\theta,\phi_j;\, \phi_{sun}) \right|^2 d\theta$ is calculated only once. The process for determining N_j, described in the previous section, may be repeated until the error is less than ε^2, where ε is the desired r.m.s. error. It is, however, difficult to specify an appropriate value of ε. Furthermore, as shown in Fig. 7, the shape of $L_\lambda(\theta,\, \phi_j;\, \phi_{sun})$ varies depending on ϕ_j and ϕ_{sun}, resulting in the variation of the mean square of L. So we normalize $E^{(\lambda)}{}_{ij}$ by the mean square of L as follows.

$$\bar{E}_{ij}^{(\lambda)} = \frac{\int_0^{\pi} \left| L(\theta,\phi_j;\, \phi_{sun}) \right|^2 d\theta - \sum_{k=0}^{i} w_{kj}^2}{\int_0^{\pi} \left| L(\theta,\phi_j;\, \phi_{sun}) \right|^2 d\theta}, \tag{13}$$

where $\bar{E}_{ij}^{(\lambda)}$ is the normalized mean square error. In our implementation, the process described in the previous section stops when $\bar{E}_{ij}^{(\lambda)}$ is less than a threshold specified by the user.

5.3 *Calculation of Sky Color Using Cosine Functions*

When the sun is in the direction of (θ_{sun}, ϕ_{sun}), the intensity of the sky in the view direction $\mathbf{v}(\theta,\phi)$ is obtained as follows.

Suppose $\phi_k \leq \phi < \phi_{k+1}$ as shown in Fig. 5. Since the table of weights is stored at sampled altitudes of the sun, weights, $w^{(\lambda)}{}_{i,k}(\phi_{sun})$ (i=0, ..., N_k) and $w^{(\lambda)}{}_{i,k+1}(\phi_{sun})$ (i=0, ..., N_{k+1}) for the arbitrary sun altitude ϕ_{sun}, are calculated by linearly interpolating using the table; this table is calculated by the method described in sections 5.1 and 5.2. Next, using Eq. (5), the intensity in the directions of (θ, ϕ_k) and (θ, ϕ_{k+1}) are calculated by the following equations.

$$L_\lambda(\theta,\phi_k) = \sum_{i=0}^{N_k} c_i w_{i,k}^{(\lambda)}(\phi_{sun}) \cos(i\theta), \tag{14}$$

$$L_\lambda(\theta,\phi_{k+1}) = \sum_{i=0}^{N_{k+1}} c_i w_{i,k+1}^{(\lambda)}(\phi_{sun}) \cos(i\theta). \tag{15}$$

Finally, the intensity of the sky in the direction (θ, ϕ) is calculated by the following linear interpolation.

$$L_\lambda(\theta,\phi) = \frac{\phi_{k+1} - \phi}{\phi_{k+1} - \phi_k} L_\lambda(\theta,\phi_k) + \frac{\phi - \phi_k}{\phi_{k+1} - \phi_k} L_\lambda(\theta,\phi_{k+1}). \tag{16}$$

5.4 *Experimental Results*

The method described in section 5.1 is used to determine the optimal number for cosine functions to represent the intensity distribution of the sky. The results are shown in this section. The altitude of the sun is set at every 1°, and the weights for each altitude of the

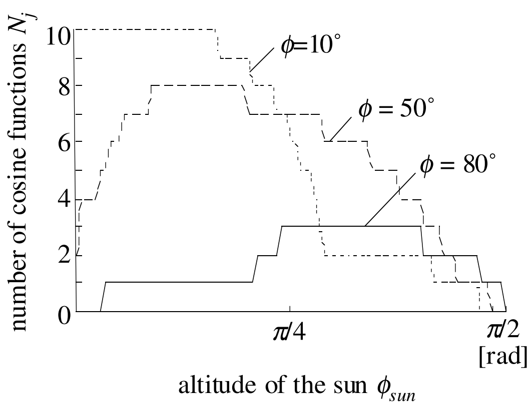

Figure 6: Relation between the number of cosine functions and the altitude of the sun.

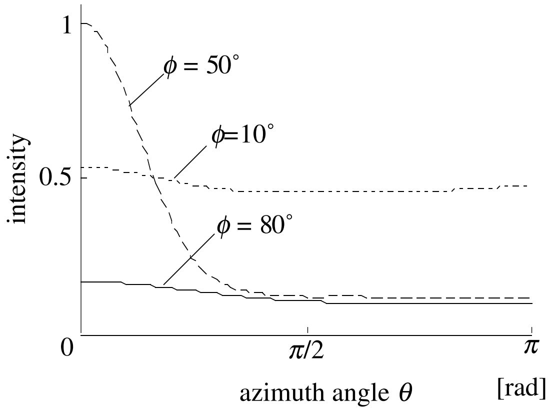

Figure 7: Approximated distribution of L. (ϕ_{sun} = 50°, λ = 675[nm])

sun are calculated by using the method described in section 5.1. The number of subdivisions of the sky dome in the direction of the ϕ, M (see Fig. 5), is set to 45 (i.e., at 2° intervals), and the number of sample points, K_w (see Eq. (10)), for calculating the weights by numerical integration is set to 90. For the error evaluation, the threshold for determining the optimal number of cosine functions is set to 1 percent. The wave length, λ, is sampled at 675, 520 and 460 [nm], figures that correspond to RGB components.

Fig. 6 shows relation between the number of cosine functions and the different altitudes of the sun. In Fig. 6, the experimental results are depicted at three elevation angles, ϕ = 10°, 40° and 80°. As shown in Fig. 6, since the change of the intensity distribution of the sky become smaller in the direction of ϕ as the sun altitude becomes higher, the higher the altitude of the sun, the smaller the number of cosine functions. Fig. 7 shows approximated distributions of $L_\lambda(\theta, \phi_j; \phi_{sun})$. ϕ_{sun} is set to 50° and each distribution is normalized by the intensity in the direction of the sun. Fig. 8 shows the intensity distribution of the sky and the relative error distribution between the intensity distribution of the sky calculated by the proposed method (see Eq. (5)) and the distribution calculated by the numerical integration method described in [Kaneda 91]. The relative error is the ratio of the difference between the approximated intensity and the true intensity to the true intensity. In Fig. 8, the sky is projected onto the xy plane and the altitude of the sun is 50°. The maximum intensity of the sky corresponds to red color in Fig. 8(a). The intensity distribution of the sky has a sharp peak around the sun as shown in Fig. 8(a). As shown in Fig. 8(b), the proposed method can calculate the intensity distribution of the sky with almost the same accuracy as the previous method [Kaneda 91]. In the proposed method, the computation time for calculating weights is 314 seconds using SiliconGraphics IRIS Indigo2, and it takes 3 seconds to calculate the intensity distribution of the sky from the weight table. The minimum number of cosine functions is 0 and the maximum is 12. The mean number of cosine functions is 5.58. Approximately 300 [KB] of memory is required for the weight table. By using method 1 described in section 4, it is necessary to divide the sky dome into 180x90 to satisfy the same accuracy. Although it takes only one second to calculate the intensity distribution of the sky in method 1, 17 [MB] of memory is required for the sky color of each grid point. In the case of method 2, spherical harmonic functions up to 20 degrees are used. But it is not enough to satisfy the same accuracy with the proposed method. Furthermore, 450 [KB] is required for the weights and it takes about one minute to calculate the intensity distribution of the sky in method 2. On the other hand, by using the proposed method, the intensity distribution of the sky for each sun altitude is stored with a reasonable memory capacity, and it is possible to calculate the intensity distribution quickly from the weight table.

6 Examples

To investigate the usefulness of the proposed method, an image of the sky is rendered by using the proposed method. The position of the sun and the viewing parameter are the same as those of Fig. 9(a), and only the sky is rendered. The size of the image is 512x384. By using the proposed method, it takes only 7 seconds to render the image pixel by pixel, while it takes 157 seconds by using the previous method [Kaneda 91]. The proposed method can display the image 22 times faster than the previous method.

Fig. 9 shows examples of natural scenes using the proposed method. Figs. 9(a) and 9(b) are images of the scenes in daytime and in the evening, respectively: The sun altitude of each image is 50° and 10°, respectively. As shown in Fig. 9, the color of the sky changes beautifully and shows drastic change when the sun altitude changes.

The proposed method was applied to architectural designs, as shown in Fig. 10. Figs. 10(a) and 10(b) are images of buildings in daytime and in the evening, respectively. The sun altitudes of Fig. 10 are the same as those of Fig. 9. In Fig. 10, clouds are rendered by using the method proposed by Gardner [Gardner 85]. As shown in Fig. 10, the sky is well reflected in the windows. The reflection of the sky in the windows is calculated by the ray tracing method [Whitted 80]. As shown in Fig. 10, the sky is beautifully reflected in the windows of the building, which results in a subtle change of the color of the windows.

The computation times of these examples are shown in Table 1. For comparison, the computation times to generate the same images using the previous method [Kaneda 91] are also shown in Table 1. By using the proposed method, images such as the sky can be generated more quickly. In particular, the examples of the natural scenes demonstrate the usefulness of the proposed method remarkably.

Table 1: Computation time.

Figure number	CPU time [sec] Proposed method	CPU time [sec] Previous method
Fig. 9(a)	81.0	356.7
Fig. 9(b)	79.8	349.5
Fig. 10(a)	650.9	1177.3
Fig. 10(b)	658.8	1136.0

7 Conclusions and Future Work

By expressing the intensity distribution of the sky by Fourier series, a fast display method of sky color is proposed. In the proposed method, by expanding the intensity distribution of the sky into a Fourier series using cosine functions and storing the weights of the series, the intensity distribution of the sky for an arbitrary position of the sun is obtained quickly. The required memory capacity to store the weights is only hundreds of kilo-bytes, not a serious problem for current workstations. Once the weights are calculated and stored as a database by using the proposed method, the color of the sky can be displayed quickly from the database.

There are several future works that remain : (1) An efficient method to reduce the calculation time of weights must be developed. (2) We propose a fast method for obtaining the intensity distribution of the sky even if the sun position changes, but the proposed method can also be easily applied to cases where the physical parameters of the atmosphere are altered. (3) Although the viewpoint is supposed to be on the ground in this paper, the height of the viewpoint other than the sun position changes in flight simulators, and the color of the sky also changes according to the height of the viewpoint.

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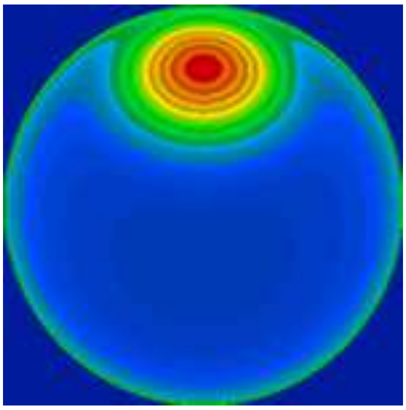
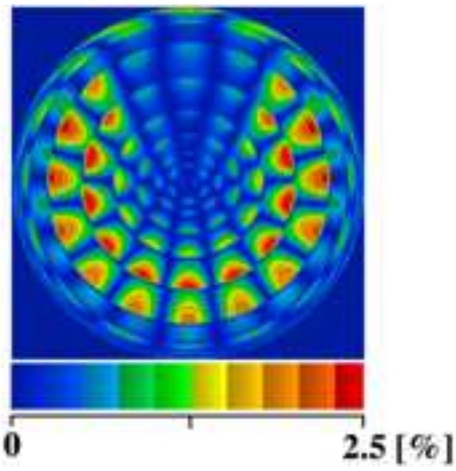
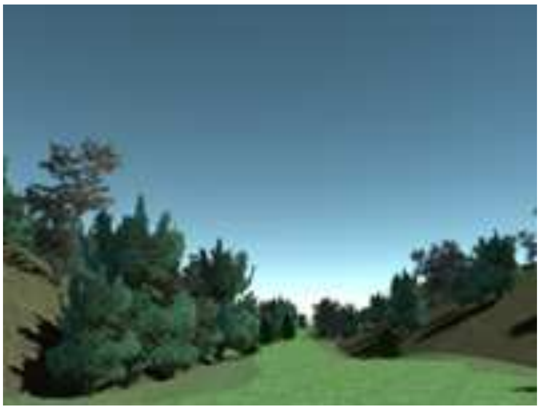


(a) The intensity distribution of the sky.

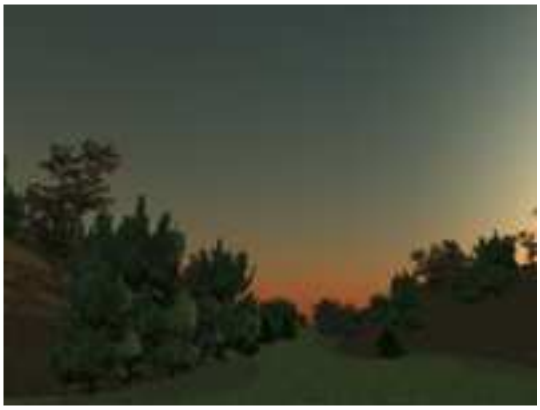


(b) Relative error distribution.

Figure 8: The intensity distribution of the sky and relative error.



(a) Daytime.



(b) Evening.

Figure 9: Examples of natural scenes.

(a) Daytime.

(b) Evening.

Figure 10: Examples of architectural designs.