A Quick Rendering Method Using Basis Functions for Interactive Lighting Design

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Abstract

When designing interior lighting effects, it is desirable to compare a variety of lighting designs involving different lighting devices and directions of light. It is, however, time-consuming to generate images with many different lighting parameters, taking interreflection into account, because all luminances must be calculated and recalculated. This makes it difficult to design lighting effects interactively. To address this problem, this paper proposes a method of quickly generating images of a given scene illustrating an interreflective environment illuminated by sources with arbitrary luminous intensity distributions. In the proposed method, the luminous intensity ditribution is expressed with basis functions. The proposed method uses a series of spherical harmonic functions as basis functions, and calculates in advance each intensity on surfaces lit by the light sources whose luminous intensity distribution are the same as the spherical harmonic functions. The proposed method makes it possible to generate images so quickly that we can change the luminous intensity distribution interactively.

Combining the proposed method with an interactive walk-through that employs intensity mapping, an interactive system for lighting design is implemented. The usefulness of the proposed method is demonstrated by its application to interactive lighting design, where many images are generated by altering lighting devices and/or direction of light.

Key words and phrases: Luminous intensity distribution, Spherical harmonic functions, Interactive lighting design, Fast rendering, Interreflective environment

1. Introduction

Recently, computer graphics have become indispensable for lighting design and interior design. Using computer graphics, designers are able to review their interior designs and lighting effects from arbitrary viewpoints. Moreover, they can easily compare many ideas by merely generating the images. In lighting design, it is very important for designers to compare lighting effects under an interreflective environment by changing lighting devices with different luminous intensity distributions and/or the direction of light. Developing such a method for interactive lighting design is desirable.

Several lighting models have been developed in computer graphics. The radiosity method, developed in 1985, can calculate luminances in a room taking into account interreflection of light between surfaces [Nishita 85, Cohen 85]. This method is an indispensable lighting model in lighting design. The radiosity method was improved upon generating more realistic images: Methods for handling not only diffuse reflection but also specular reflection and for taking into account the scattering of light due to particles in the air were developed [Immel 86, Rushmeier 87, Sillion 91].

With these methods, photorealistic images can be generated. One problem that arises, however, is that computation time invariably increases. To address the problem, several methods for speeding up the radiosity calculation were proposed [Cohen 88, Shao 88, Chen 90]. It turned out, however, that designers could not get rendered images fast enough to interactively design lighting effects in an interreflective environment, even when employing these methods. To generate an image quickly under a fixed lighting condition, a method of luminance mapping with graphics hardware was developed [Akeley 88, Baum 89]. In the method, luminances at sampled points on surfaces are calculated in advance, and an image is generated in real time by interpolating with these luminances using a graphics hardware. With this method, images from different view points and/or view reference points were obtained in real time. The method cannot generate, however, images in real time with different luminous intensity distributions of lights and/or direction of light, which are indispensable for interactive lighting design.

This paper proposes a method of generating images, taking into account interreflection of light, fast enough for interactive lighting design when luminous intensity distributions and/or direction of light are changed. In the proposed method, luminous intensity distributions are decomposed into a series of basis functions, and luminances are obtained by simply summing each luminance from light sources whose luminous intensity distributions obey each basis function. In other words, pre-processing luminances on surfaces are calculated for each light source whose luminous intensity distribution obeys a basis function. When luminous intensity distributions of lights are specified, weights for basis functions are calculated to express the specified distributions. Resulting images are obtained very quickly by summing the pre-calculated luminances multiplied by the corresponding weights.

The proposed method uses a series of basis functions for representing the luminous intensity distribution of light. Kajiya et al. used spherical harmonic functions to represent the phase function of particles in clouds [Kajiya 84]. Both Cabral et al. and Sillion et al. used spherical harmonic functions to represent the reflectance functions on surfaces [Cabral 87, Sillion 91].

The proposed method makes it possible to generate images very quickly even when luminous intensity distributions and/or direction of light are altered, as only the weights for each basis function are updated. Thus, images are generated simply by changing the weights of pre-calculated intensities on the surface and summing those intensities with corresponding weights. The method proves useful for interactive lighting design under an interreflective environment, especially when designing lighting effects from light sources with directional characteristics, such as a spotlight.

2. Basic Idea

Assuming multiple light sources, luminance at a certain point is obtained by calculating the luminance from each light source and summing them. This means linearity is maintained in the calculation of luminance [Kajiya 86, Schoenenman 93, Nimeroff 94]. In general, luminance calculation obeys the two following properties [Nimeroff 94].

1. Luminance from two light sources is equivalent to the sum of luminances from the individual light sources.
2. Multiplying the luminance from a light source by a factor w is equivalent to multiplying the intensity of the light source by w.

Nimeroff et al. applied this linearity of luminance calculation to calculating natural luminance from skylight [Nimeroff 94]. They proposed an efficient method of re-calculating luminance even when the intensity distribution of sky light or the sun position is altered. Schoenenman et al. employed these properties to solve lighting design as an inverse problem [Schoenenman 93].

In this paper, we make use of these properties to calculate luminance in a room, taking into account the interreflection of light. We propose a quick method of calculation of luminance in the case of alterations in the luminous intensity distributions and the direction of light sources. Furthermore, the proposed method makes it possible to walk through the scene interactively.

In the proposed method, the luminous intensity distribution of a point light source is expressed as the sum of a series of basic distributions, as shown in Fig. 1. Luminance due to a light source whose luminous intensity distribution corresponds to one of the basic distributions is calculated in advance, and stored as

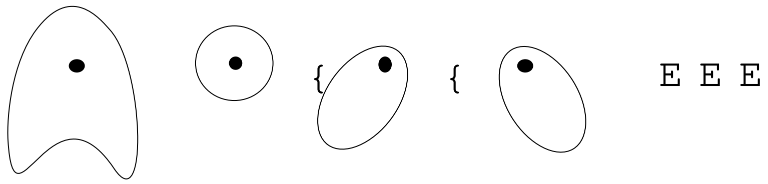

Figure 1: Luminous intensity distribution and its basis components.

basis luminance. Using the aforementioned property 1, the luminance due to the light source with the luminous intensity distribution shown in the left side of Fig. 1 is calculated by summing the pre-calculated basis luminances corresponding to each individual basic distribution. Moreover, using property 2, the luminance due to the light source whose luminous intensity distribution can be expressed as the weighted sum of the basic distributions, shown in the right side of Fig. 1, is obtained by multiplying each basis luminance with the corresponding weights and summing them. Thus, once the basis luminance corresponding to each basic distribution is calculated in the pre-process, the resulting luminance can be obtained quickly merely by calculating the weighted sum of the basis luminances.

In the proposed method, surfaces comprising a scene are divided into small patches and the basis luminances corresponding to each basic distribution are stored at the vertices of these patches. The method makes it possible to generate images interactively from different view points and/or reference points.

3. Decomposing Luminous Intensity Distribution

3.1 Approximation of Luminous Intensity Distributions for Axisymmetrical Light Sources

Luminous intensity distributions can be expressed by using a series of basis functions that are orthogonal to each other, as vectors in 3D space are expressed by using three orthogonal unit vectors. In the case of axi-symmetrical luminous intensity distribution, such as point light sources, the luminous intensity distribution depends on only the angle, γ, from the direction of light (see Fig. 2(b)). Thus, the luminous intensity distribution $I_{light}(\gamma)$ is expressed by the following equation.

$$I_{light}(\gamma) = \sum_{l=0}^{N} w_l \varphi_l(\gamma), \tag{1}$$

where $\varphi_l(\gamma)$ is a basis function, w_l is a weight for each basis function, and N is a number of basis functions necessary to express the luminous intensity distribution. Equation 1 implies that luminous intensity distribution can be controlled by changing weights for each basis function. This is similar to a mathematical expression of curves and surfaces using basis functions, such as Bézier curves and surfaces.

The selection of basis functions is an important factor for expressing wide varieties of distributions. We use Legendre polynomials as a basis function, as light sources with different directions of light can be expressed by using spherical harmonic functions, as described in Section 3.2.

Using Legendre polynomials in degree l, $P_l(\cos\gamma)$, as basis functions, luminous intensity distribution is expressed by the following equation.

$$I_{light}(\gamma) = \sum_{l=0}^{N} w_l P_l(\cos\gamma). \tag{2}$$

From the property of the orthogonal functions, the weight, w_l, for the Legendre function in degree l is calculated by the following equation.

$$w_l = \frac{2l+1}{2} \int_0^{\pi} I_{light}(\gamma) P_l(\cos\gamma) \sin\gamma \, d\gamma. \tag{3}$$

When the luminous intensity distribution of a light source, $I_{light}(\gamma)$, is specified, the weights for Legendre polynomials are calculated by Eq. 3. The specified luminous intensity distribution is then approximated by using Eq. 2. If the number of Legendre polynomials in Eq. 2 is sufficient, the approximated distribution becomes very close to the specified distribution.

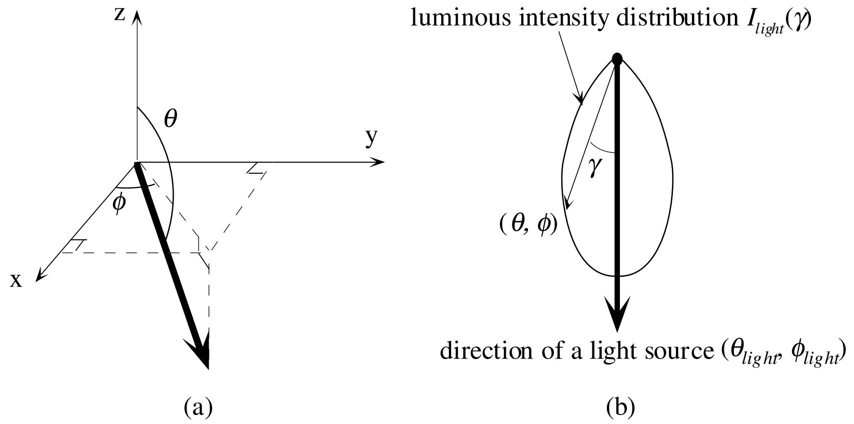

Figure 2: The luminous intensity distribution of a light source.

3.2 Changing Direction of Light

If the direction of light does not need to be changed, Legendre polynomials is sufficient for representing the luminous intensity distribution of the light source. However, in interior lighting design, it is often necessary to change the direction of light. To change direction, spherical harmonic functions are introduced. Thus, when changing the direction of light, the luminous intensity distribution expressed by Eq. 2 can be replaced by using spherical harmonic functions. We express an arbitrary direction viewed from the position of the light source with the polar coordinates (θ, ϕ), where θ is an angle from the z axis and ϕ is an angle from the x axis as shown in the Fig. 2(a). Assuming that the direction of light is $(\theta_{light}, \phi_{light})$, and the direction (θ, ϕ) has angle γ from the direction of light (see Fig. 2(b)), the luminous intensity distribution expressed by Eq. 2 is rewritten by the following equation.

$$I_{light}(\gamma) = \sum_{l=0}^{N} w_l P_l(\boldsymbol{r}_{light} \cdot \boldsymbol{r}), \tag{4}$$

where $\boldsymbol{r}_{light}$ and $\boldsymbol{r}$, respectively, are unit vectors toward the direction of light and the direction whose angle is γ from the direction of light, and (•) is the dot product.

$$\boldsymbol{r}_{light} = (\sin\theta_{light}\cos\phi_{light}, \sin\theta_{light}\sin\phi_{light}, \cos\theta_{light}), \tag{5}$$

$$\boldsymbol{r} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta). \tag{6}$$

By the addition theorem of spherical harmonic functions, the following equation is introduced [Hobson 55].

$$P_l(\boldsymbol{r}_{light} \cdot \boldsymbol{r}) = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}(\theta_{light}, \phi_{light}) Y_{lm}(\theta, \phi), \tag{7}$$

where $Y_{lm}(\theta, \phi)$ is a spherical harmonic function (see Appendix).

From Eqs. 4 and 7, the luminous intensity distribution whose direction of light is $(\theta_{light}, \phi_{light})$ is expressed by the following equation.

$$I_{light}(\theta, \phi;\, \theta_{light}, \phi_{light}) = \sum_{l=0}^{N} \frac{4\pi}{2l+1} w_l \sum_{m=-l}^{l} Y_{lm}(\theta_{light}, \phi_{light}) Y_{lm}(\theta, \phi) \tag{8}$$

$$= \sum_{l=0}^{N} w_l \sum_{m=-l}^{l} A_{lm}(\theta_{light}, \phi_{light}) Y_{lm}(\theta, \phi), \tag{9}$$

where

$$A_{lm}(\theta_{light}, \phi_{light}) = \frac{4\pi}{2l+1} Y_{lm}(\theta_{light}, \phi_{light}). \tag{10}$$

Equations 9 and 10 imply that when changing the direction of light, the resulting image is generated by the following procedure. First, basis luminances corresponding to each spherical harmonic function are calculated. When the luminous intensity distribution of the light source is specified, weights, w_l, for each Legendre polynomial are calculated by using Eq. 3. Next, when the direction of light is specified, then weights, A_{lm}, for each spherical harmonic function are calculated by using Eq. 10. The luminance due to luminous intensity distribution is obtained by multiplying basis luminances by the corresponding weights and summing them. Note that from Eq. 9, $(N+1)^2$ terms of spherical harmonic functions are required to change the direction of light expressed by using Legendre polynomials up to degree N.

The color of the light source can also be handled using the proposed method. First, assuming the spectral distribution of the light source is uniform, basis luminances for each spectral component, such as RGB, are pre-computed. Luminances for each spectral component at a certain point are obtained by employing the proposed method for each spectral component. Then, the luminances for each spectral component are multiplied by the corresponding component ratios of the colored light source.

3.3 Spotlight and Sigma Factor

In this section, a problem in representing a spotlight using a series of basis functions is discussed, and a method of overcoming this problem is described.

3.3.1 Gibbs Phenomenon

Generally, the luminous intensity distribution of a spotlight is expressed by the following equation.

$$I_{light}(\gamma) = \begin{cases} \cos^k\gamma & 0 < \gamma < \gamma_{spread} \\ 0 & \gamma_{spread} < \gamma < \pi \end{cases}, \tag{11}$$

where, γ is the angle from the direction of light, γ_{spread} is the spread angle of the spotlight, and k express the sharpness of the spot. Expanding the luminous intensity distribution expressed by Eq. 11 into a series of Legendre polynomials, infinite terms of Legendre polynomials are required. The distribution is approximated by a finite set of Legendre polynomials because an infinite number of Legendre polynomials cannot be handled in practical use. This causes what is known as the Gibbs phenomenon, where undesirable ripples appear as a serious approximation error. The approximate distribution for the angle $\gamma_{spread} < \gamma < \pi$ does not equal zero, though the original distribution does equal zero.

For example, suppose k=10, $\gamma_{spread}=\pi/2$ in Eq. 11 and approximate it by a set of Legendre polynomials with degree up to 8. Figure 3 (a) shows a rendered image lit by a light source with the approximated luminous intensity distribution. The light source is placed at the center of the ceiling and its luminous intensity distribution is also shown. Bright artificial circles appear around the light source due to the ripples of the Gibbs phenomenon.

When approximating a narrow-beamed light source using a finite set of basis functions, the Gibbs phenomenon becomes a serious problem. To address this problem and suppress the ripples, we introduce a sigma factor, which is often employed in the field of digital filtering.

3.3.2 Introducing Sigma Factor

The ripples caused by approximating a distribution with a finite set of orthogonal basis functions up to degree N have the same frequency as that of the basis function in degree N+1 [Hamming 77]. As for the example given in the previous section, ripples have the same frequency as that of Legendre polynomials of degree 9. By integrating the approximated distribution between the period, the effect of the ripples can be suppressed [Hamming 77]. Applying this idea, each basis function is multiplied by sigma factor, $\sigma(N, l)$, expressed by Eq. 13, and the luminous intensity distribution is expressed by the following equation.

$$I_{light}(\gamma) = \sum_{l=0}^{N} \sigma(N,l) w_l P_l(\cos\gamma), \tag{12}$$

$$\sigma(N,l) = \begin{cases} 1.0, & l = 0 \\ \dfrac{\sin(\pi l/(N+1))}{\pi l/(N+1)}, & l = 1, 2, \cdots, N \end{cases}, \tag{13}$$

where w_l is calculated by Eq. 3. Figure 3 (b) shows an image calculated by using the sigma factor, where the scene is lit by a spotlight with the distribution of Eq. 11. By employing the sigma factor, the effects of the ripples are suppressed and the bright artificial circles disappear.

The sigma factor is not needed for spherical harmonic functions because rotation can be exactly expressed by spherical harmonics functions. That is, once the effects of the ripples are suppressed by applying the

Figure 3: Suppressing ripples using a sigma factor.

sigma factor to Legendre polynomials, ripples no longer occur in the rotation of the distribution resulting from direction.

3.4 Calculation of Basis Luminances

Basis luminances are calculated prior to generating images. Which is to say, surface luminances are calculated for each light source whose luminous intensity distribution obeys a spherical harmonic function expressed by Eq. 9. In the proposed method, only the luminous intensity distributions of light sources are expressed with spherical harmonic functions. All surfaces are assumed to be purely diffuse. This process could be carried out by repeating radiosity calculation $(N+1)^2$ (N is the number of basis functions required) times. The calculation time would proportionally increase to the number of basis functions, taking a great deal of time. To solve this problem, the calculation of common parts in the basis luminance computation, such as form factor calculation, should be carried out only once.

Assuming the number of patches in the scene is n_{patch}, the radiosity equation is expressed by the following equation [Cohen 85].

$$B_i = E_i + \rho_i \sum_{\substack{j=1 \\ i \neq j}}^{n_{patch}} F_{ij} B_j, \quad i = 1, .., n_{patch}, \qquad (14)$$

where B_i, E_i and ρ_i are radiosity, emission, and reflectivity of patch i, respectively, and where F_{ij} is a form factor between patches i and j. To deal with point light sources, we use luminance of patch i due to direct light as self-emission E_i. Assuming lit by a light source with the luminous intensity distribution of a spherical harmonic function, $Y_{lm}(\theta, \phi)$, which is expressed by Eq. 22, the luminance, E_i, due to the direct light is expressed as follows.

$$E_i^{(lm)} = G_i Y_{lm}(\theta_i, \phi_i), \tag{15}$$

where G_i is a factor determined by the geometry between the light source and patch i , and (θ_i, ϕ_i) is the direction of patch i viewed from the light source. Inserting Eq. 15 into Eq. 14, the following equation is obtained.

$$B_i^{(lm)} = G_i Y_{lm}(\theta_i, \phi_i) + \rho_i \sum_{\substack{j=1 \\ i \neq j}}^{n_{patch}} F_{ij} B_j^{(lm)}, \quad i = 1, .., n_{patch}. \tag{16}$$

Basis luminances for each spherical harmonic functions are obtained by solving the following equation.

$$\begin{bmatrix} B_i^{(00)} \\ B_i^{(1,-1)} \\ \vdots \\ B_i^{(NN)} \end{bmatrix} = G_i \begin{bmatrix} Y_{00}(\theta_i, \phi_i) \\ Y_{1,-1}(\theta_i, \phi_i) \\ \vdots \\ Y_{NN}(\theta_i, \phi_i) \end{bmatrix} + \rho_i \sum_{\substack{j=1 \\ i \neq j}}^{n_{patch}} F_{ij} \begin{bmatrix} B_j^{(00)} \\ B_j^{(1,-1)} \\ \vdots \\ B_j^{(NN)} \end{bmatrix}, \quad i = 1, .., n_{patch}. \tag{17}$$

From Eq. 17, it is clear that G_i and F_{ij} are common factors between different basis luminances. In the proposed method, these factors are calculated only once to decrease the computation time of the basis luminances.

3.5 Generating Images from Basis Luminances

In the pre-process, basis luminances for each spherical harmonic function are calculated and stored at vertices of each patch. Using basis luminances, images are generated by the following procedure:

(0: pre-process) Geometrical factors G_i, F_{ij} are computed, and Eqs. 16 and 17 for basis luminances, $B_i^{(lm)}$ are solved.

(1) The luminous intensity distribution and direction of the light source are specified.

(2) From Eqs. 3 and 10, weights w_l and A_{lm} are calculated.

(3) Luminances, B_i, at the vertices of each patch, i, are calculated by multiplying basis luminances, $B_i^{(lm)}$, by the corresponding weights obtained in step 2, and summing them.

$$B_i = \sum_{l=0}^{N} \sigma(N,l) w_l \sum_{m=-l}^{l} A_{lm}(\theta_{light}, \phi_{light}) B_i^{(lm)}. \tag{18}$$

(4) From the luminance obtained in step 3, an image with a specified view point and a view reference point is generated by using luminance mapping.

In lighting design, various images with different view points and/or reference points can be quickly generated by repeating steps 1 through 4.

4. Interactive Lighting Design System

In this section, an interactive system to design lighting effects using the proposed method is discussed. The proposed system makes it possible to generate images with different camera positions, while changing the luminous intensity distributions of light sources interactively. We propose the two graphical user interfaces described below to edit the luminous intensity distribution and the direction of the light source.

4.1 Editing Luminous Intensity Distribution

The proposed user interface to edit the luminous intensity distribution is shown in Fig. 4. Several control points are provided to edit the distribution curve. To calculate the weight in Eq. 3, I_{light} is obtained by linearly interpolating between these control points. In the editing window, an approximated luminous intensity distribution from Legendre polynomials is also displayed. Every time a control point is moved, the weights are calculated for each Legendre polynomial. A method of calculating the weights incrementally is employed so that the distribution can be edited interactively. In the incremental method, each weight is updated by calculating the increment from the movement of a control point.

Equation 3 is descretized by using Simpson's integral formula,

$$w_l = \frac{2l+1}{2}\left\{ \frac{1}{2}(I_{light}(0)\,f_l(0) + I_{light}(\pi)\,f_l(\pi)) + \sum_{i=1}^{M-1} I_{light}(i\Delta\gamma)\,f_l(i\Delta\gamma) \right\}\Delta\gamma, \tag{19}$$

where,

$$f_l(\gamma) = P_l(\cos\gamma)\sin\gamma, \tag{20}$$

and M+1 is the number of sample points for the numerical integration, and $\Delta\gamma$ is the interval between sample points. For simplicity, suppose the number of control points is equal to that of the sample points, M+1. Moving the k-th control point, the updated value of the weight , $w_l^{(new)}$, is calculated by the following equation.

$$\begin{aligned} w_l^{(new)} &= w_l - t_k(I_{light}(k\Delta\gamma) - I_{light}^{(new)}(k\Delta\gamma))f_l(k\Delta\gamma)\Delta\gamma, \\ t_k &= \begin{cases} 1/2, & k=0 \text{ or } k=M \\ 1 & \text{otherwise.} \end{cases} \end{aligned} \tag{21}$$

where $I_{light}^{(new)}(\gamma)$ is the luminous intensity distribution after moving the k-th control point. By using Eq. 21, the weights are incrementally calculated when the control point is moved. Furthermore, $f(\gamma)$ to each sample point can be stored as a table for speeding up the calculation. These techniques make it possible to edit the luminous intensity distribution interactively.

4.2 Specifying Light Source Direction

A simple and straight-forward way to specify the direction of a light source is to use the polar coordinate system, whose origin is located at the light source. In many cases, however, it is convenient to specify the point in a room that the direction of the light source is oriented to. The following method is employed to specify the direction of light in the proposed system. When a point on the screen is specified, the three-dimensional coordinate of the point is calculated by using inverse perspective transformation, and the direction of a light source is oriented to the point.

To specify the direction of the light interactively, the patch including the point specified on the screen must be found in real time. To achieve this, we employ the method proposed by Hanrahan et al. [Hanrahan 90]. In other words, together with the usual frame buffer to store the pixel's color, a specific buffer is prepared to store the patch's ID corresponding to each pixel. Using the specific buffer, the patch's ID at the specified point can be obtained immediately. Using inverse perspective transformation, the three-dimensional coordinate of the specified point is then calculated. Figure 5 shows the user interface for specifying the direction of the light source.

4.3 Generating Images

As described in the previous sections, the user can edit the luminous intensity distribution and change the direction of a light source interactively. A message to render the image is then sent to the system. Receiving the message, the system calculates luminances at vertices of patches by using the basis luminances and renders the patches by employing Gouraud shading. Employing graphics hardware, the user can walk through the scene with an arbitrary view point or view reference point. The user can examine the lighting effects interactively by displaying images with various light sources and different camera positions.

5. Examples

To investigate the usefulness of the proposed method, we applied the method to a simple model of a room and generated images using different luminous intensity distributions of light sources and different directions of light. Figure 6 (a) shows an image rendered by the traditional method [Nishita 85]. There are two light sources on the ceiling, one for lighting the entire room and the other for lighting a local part of the room. Luminous intensity distributions of each light source are shown in the figure. Figure 6 (b) shows an image

Table 1 : Computation time.

	pre-process	rendering
Figure 6 (a)	-	64.6 [min.]
Figure 6 (b)	180 [min.]	1.2 [sec.]

machine : SGI IRIS Indigo2

rendered by the proposed method. Luminous intensity distributions of light sources located at the center and the corner of the ceiling are expressed by a set of spherical harmonic functions in degrees up to 5 and 8, respectively, resulting in 36 and 81 terms of spherical harmonic functions, respectively. The luminous intensity distributions are also shown in the figure. Surfaces in the scene are divided into 16,000 patches for radiosity calculation, but the patches are not subdivided adaptively in this figure. Figure 6 (c) shows an intensity error distribution between the images Figs. 6 (a) and 6 (b). The maximum intensity error is about 20 %, but the area is small as shown in Fig. 6 (c). Comparing Figs. 6 (a) with 6 (b), it is clear that the image generated by the proposed method is nearly the same quality as the image using the traditional method. All images were rendered on a SiliconGraphics IRIS Indigo2. The computation time is shown in Table 1. The luminous intensity distribution of the light source at the center of the ceiling and the direction of the light source at the corner of the ceiling are altered (see Fig. 6 (d)). In this figure, the view point is also changed. The rendering time of Fig. 6 (d) is the same as that of the image shown in Fig. 6 (b). These examples show that the proposed method can generate the images quickly enough to interactively design the lighting effects with different lighting parameters, such as luminous intensity distribution and direction of light.

The proposed method was applied to lighting design of a living room as shown in Fig. 7. There are two light sources on the ceiling and one more light at the corner of the room for indirect lighting. In Fig. 7 (a), all the light sources are turned on brightly and the color of the light sources is white. Each luminous intensity distribution of the lights is also shown in Fig. 7 (a). In Fig. 7 (b), the intensity of the light sources on the ceiling is weaken, while the intensity of the indirect light is strengthen. The color of the light sources is set to warm colors. Luminous intensity distributions of each light source are also shown in Fig. 7(b).

All images are generated on the same machine as Fig. 6, and it took 2.5 seconds to render each image. Using the proposed method, designers can compare the lighting effects interactively by editing the luminance intensity distributions of light sources and the color of light.

6. Conclusions and Future Works

By expressing luminous intensity distribution of a light source with basis functions, the proposed method makes it possible to generate images very quickly when luminous intensity distribution and/or direction of light is modified. In the proposed method, basis luminances at each vertex of patches are calculated in advance. Images with arbitrary luminous intensity distributions are then generated by multiplying the basis luminances by the corresponding weights and summing them. It is therefore possible to generate the images sufficiently quickly for interactive lighting design. Combining the proposed method with intensity mapping, images are generated very quickly, even if both the lighting parameters and view parameters are changed.

As for the future work that remained, to approximate a complicated luminous intensity distribution, the number of terms of basis functions increases. This necessitates an increase in the size of the memory to store the basis luminances. A method of data compression to decrease the size of the basis luminances must be investigated.

It is possible to extend the proposed method to inverse problems of lighting design. Given a desired luminance for each patch, an inverse problem gives the weights for each basis function that satisfy the desired luminance for determining the luminous intensity distribution of the light. Using the proposed method, lighting designers can visualize the result of the inverse lighting problem immediately and can also modify the resulting luminous intensity distribution of the light source interactively. The approach makes it possible to design lighting effects from both direct and inverse approaches.

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Appendix

Spherical harmonic functions, $Y_{lm}(\theta,\phi)$, are expressed by the following equation [Hobson 55].

$$Y_{lm}(\theta,\phi)=\begin{cases}(-1)^m\sqrt{\dfrac{2l+1}{4\pi}\dfrac{(l-m)!}{(l+m)!}}\,P_{lm}(\cos\theta)\cos(m\phi), & m\geq 0\\ (-1)^m\sqrt{\dfrac{2l+1}{4\pi}\dfrac{(l-m)!}{(l+m)!}}\,P_{lm}(\cos\theta)\sin(m\phi), & m<0\end{cases} \tag{22}$$

where $P_{lm}(x)$ is associated Legendre polynomials. Associated Legendre polynomials are evaluated with the following recurrence relations.

$$P_{lm}(x)=(2l-1)\sqrt{1-x^2}P_{l-1,m-1}(x)+P_{l-2,m}(x), \tag{23}$$

where $P_{00}(x)=1$ and $P_{10}(x)=x$. Here, $P_{l0}(x)$ is equal to Legendre polynomials, $P_l(x)$ [Hobson 55].

Figure 4: The proposed user interface for editing luminous intensity distribution.

Figure 5: Specifying the direction of the light source.

(a) The traditional method.

(c) Error distribution between Fig. 6(a) and Fig. 6(b).

Figure 6: Lighting design of a simple room.

(b) The proposed method.

(d) Changing luminous intensity distributions.

Figure 6: Lighting design of a simple room.

(a) A brightly illuminated room.

(b) Changing luminous intensity distributions and color of light.

Figure 7: Lighting design of a living room.