HALF-TONE REPRESENTATION OF 3-D OBJECTS ILLUMINATED BY AREA SOURCES OR POLYHEDRON SOURCES

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Abstract

The degree of rearism of the shaded image of a three-dimensional scene depends remarkably on the successful simulation of shadowing and shading effects. The shading model has two main ingredients: properties of surface and properties of the illuminations falling on it. In most previous work, it seems that researchers' interest has been concentrated on the former rather than the latter, and a major deficiency in most computer-synthesized images has been the lack of penumbrae.

This paper presents shading algorithms for area sources and polyhedron sources. The advanced points are as follows: 1) The use of shadow volumes formed by a convex polyhedron and an area (or polyhedron) source results in easy determination of regions of penumbrae and umbrae on faces. 2) The illuminance in penumbrae caused by several polyhedra can be obtained by using the contour integration method. 3) The precise calculation of the illuminance for area sources and polyhedron sources gives the much-improved reality of half-tone representation.

1. Introduction

Half-tone representation of three-dimensional objects is one of the useful tools not only for CAD/CAM of buildings and machines but also for lighting problems. This paper discusses the depiction method of quite realistic images of scenes useful for lighting design.

In order to display three-dimensional objects lending reality and vividness, researchers have recently developed expression techniques considering the properties of objects such as reflection, refraction and/or transparency(e.g., [1,2,3]). However, the light sources used in those techniques have been limited to parallel light sources and/or point light sources. Therefore, most previous algorithms have handled umbrae only. Actual artificial sources have their finite sizes and their shadows involve umbrae and penumbrae. In many cases of the lighting design of room interiors, the illumination from windows and/or a luminous ceiling should be handled as the illumination from area sources.

This paper proposes the methods of half-tone representation of three-dimentional objects illuminated by area sources or polyhedron sources. Objects are treated as sets of convex polyhedra, and the shapes of light sources are convex polygons or convex polyhedra; the intensity characteristics of them are the lambertian distribution.

2. Shading for Area Sources and Polyhedron Sources

Calculating illuminance at each point on 3-D objects is necessary for half-tone representation of these objects, and requires the shadow detection of each point. The method, detecting the shadow boundaries on each face prior to illuminance calculation in order to shorten the computation time, has been proposed[4],[5]. This paper makes use of the same idea, that is, shadow detection and calculation of shadow boundaries are done prior to illuminance calculation.

The conception of shadow volume for point sources and parallel sources has been proposed[6]. For its improvement, we propose an umbra volume and a penumbra volume formed by a convex polyhedron and an area(or polyhedron) source. Furthermore, we discuss the calculation method of shadow boundaries on each face by using these shadow volumes as well as the method of illuminance calculation in penumbra regions.

2.1 Preparation

For the facilitation of the following discussion, some definition and functions are given prior to the main subject:

(1) 3-D objects are treated as sets of convex polyhedra and convex polygons. Each face of a polyhedron is defined by the string of vertices in a clockwise direction when viewed from outside the polyhedron, and the normal of a face is assumed as an outward pointing vector.

(2) The function F which decides the relationship between a face and a point, and the function H which decides the relationship between a face and a polyhedron(or a face), are defined as follows:

The relationship between a face S_j consisting of vertices P_i (i=1,2,3...) and an arbitrary point Q(X,Y,Z) is determined by the following equation:

$$F_{S_j}(Q)=(P_3-P_1)\times(P_2-P_1)\cdot(Q-P_1)=a_jX+b_jY+c_jZ+d_j. \quad (1)$$

where (a_j, b_j, c_j) is the normal of S_j. Assuming that the half-space of the pointing side of the normal of S_j is positive-space and the rest of 3-D space is negative-space, and if $F_{S_j}(Q)>0$, then Q exists within the positive-space. If $F_{S_j}(Q)<0$, then Q exists within the negative-space. If $F_{S_j}(Q)=0$, then Q exists on the plane including S_j.

For a polyhedron V(or a face S) consisting of vertices P_k(k=1,2,...,n;n=number of vertices of V or S), H is defined by the following equation:

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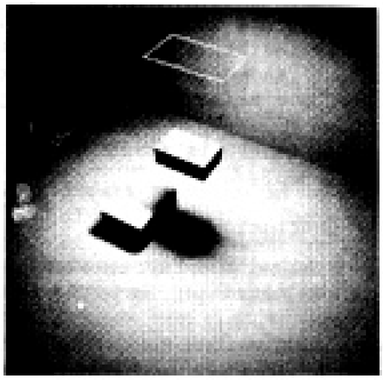

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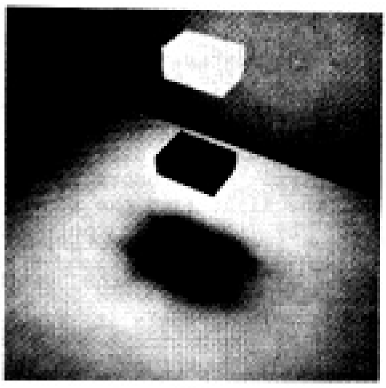

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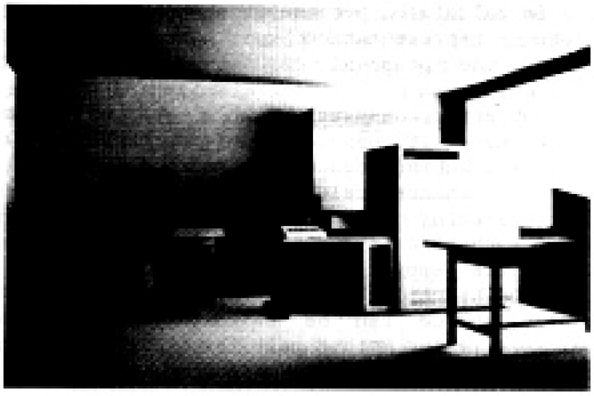

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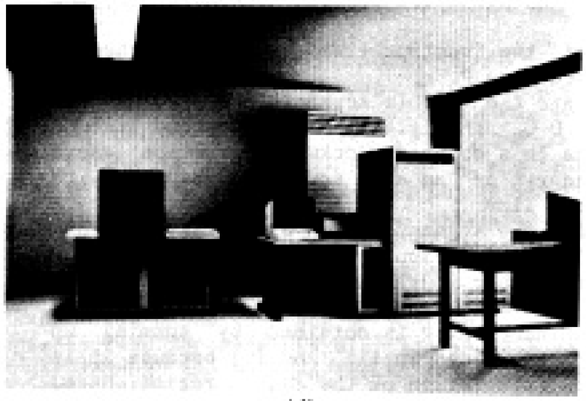

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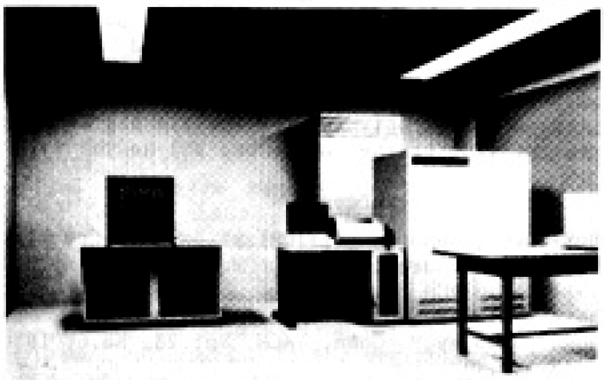

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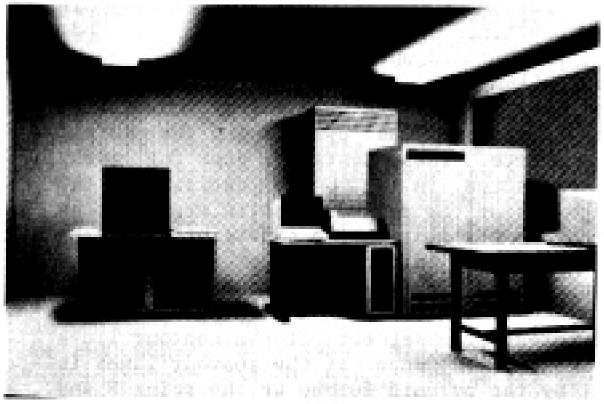

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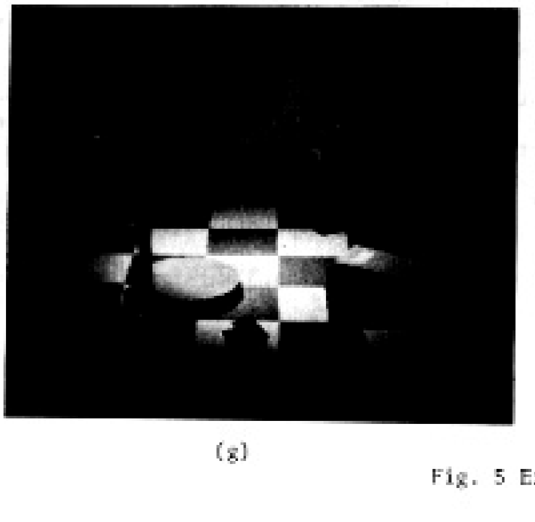

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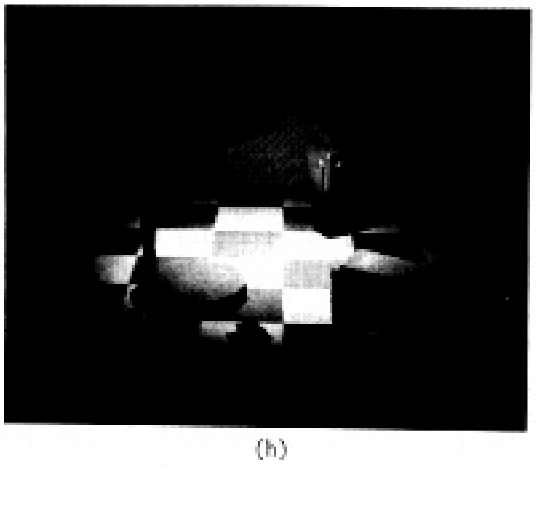

(h)

Fig. 5 Examples.

(2) Illuminance Calculation in Shadow Areas

For umbrae, the illuminance calculation is simplyfied. Where we consider the shadows from many polyhedra, if the point P is included in at least one umbra, then the illuminance at P is zero.

On the other hand, the illuminance calculation in the penumbra is more complex. A penumbra area is the region in which the light from the source is partially interrupted by several polyhedra. Therefore, the illuminance at the point P must be calculated by applying equation (7) for the visible parts of the source from P.

The illuminance in the penumbra caused by several polyhedra can be calculated by the following procedure:

i) Extract the polyhedra casting penumbra on the point P.
ii) Extract the contour lines of those polyhedra when viewed from P.
iii) Integrate the visible segments(e.g., Q_rQ_s in Fig.4-a, Q_lQ_t in Fig.4-b) of the contour line of the source in a counter-clockwise direction where the boundary of an area source corresponds with the contour line.
iv) Integrate the segments which exist within the contour line of the source and outside the contour lines of another polyhedra (e.g., P_1P_t in Fig. 4-a), in a clockwise direction.

The illuminance at P is obtained by summing up the integrated values in iii) and iv) because it is equal to the integration of the closed region(shaded regions in Fig. 4).

The integration segments(visible segments) in iii) are determined by the following method: Considering the pyramid(open convex space) formed by the point P and the contour line of the convex polyhedron, the segments are obtained by the intersection test between the pyramid and the contour line of the source. These tests are done for every polyhedra extracted in i). The decision of the visible segments of the contour line is executed by using the notion of quantitative invisibility used for hidden line ellimination[7,8]; the segments whose quantitative invisibility equals zero are visible.

For the integration segments of the contour lines in iv), first each edge of the contour lines is clipped out by the pyramid formed by the point P and the contour line of the source; outside segments of the pyramid are cut off. Then the integration segments are obtained in the same manner as iii).

3. Examples

Some examples for the proposed method are shown in Fig. 5.

Picture (a) is an example of a rectangular surface source and (b) is an example of a rectangular prism source.

Pictures (c) through (f) are examples of practical application for lighting simulation of the interior of a room-the computer room in which these examples were made. Pictures (c) and (d) show the room in the day time; in the former the room is illuminated only by a window, while in the latter a rectangular source is lighted. Pictures (e) and (f) show the examples of the room in the night; the former has two rectangular surface sources and the latter has two rectangular prism sources.

Picture (g) depicts an example of a sphere source. Picture (h), where the light source is assumed as a point source, is shown in order to compare the lighting effect; the effect of penumbrae gives much reality.

The computer used here is TOSBAC DS-600, and the CRT is GRAPHICA M-508.

4. Conclusion

This paper described the representation method of 3-D objects illuminated by area sources or polyhedron sources.

The following conclusions can be stated from the results:

1) The illuminance for area sources or polyhedron sources is calculated precisely, and the reality of half-tone representation is much improved. Therefore, the proposal can be applicable to lighting designs.
2) 3-D objects composed with a set of convex polyhedra make it easy to obtain the volumes of penumbrae and umbrae needed for shadow detection.
3) The illuminance calculation becomes simple because searching for the areas of penumbrae and umbrae on each face is completed before scanning for hidden surface removal.
4) The illuminance in unshadowed portions for a polyhedron source can be calculated by using the contour integration method only for the contour line of the source when viewed from the calculation point.
5) The illuminance calculation in the penumbra caused by several polyhedra is done by setting the visible parts of the source when viewed from the calculating point as the new sources. Therefore, the illuminance can be obtained by using the contour integration method only for the visible segments of the contour line of the source and polyhedra.

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as for both area sources and polyhedron sources. That is, the shadow exists on the face intersecting the penumbra or the umbra volume as previously mentioned.

In the first step, an intersection test between a face and a penumbra volume is executed because a penumbra volume always includes an umbra volume. If the penumbra volume and the face intersect each other, there is at least one penumbra on the face. In this case, the umbra is detected by means of the intersection test between the umbra volume and the face; where the faces required for shadow detection are only the faces of type A and B.

We describe the decision method whether or not a polyhedron V casts a shadow on a face S_f for a source S_e(or V_e). Assume that $\bar{U}$ is the penumbra volume formed by the source Se(or Ve) and the convex polyhedron V, and $\bar{S}p(p=1,2,\dots)$ are the faces of $\bar{U}$, then there is a possibility of intersection only when the equations (5) and (6) are satisfied.

$$H(S_f,V)\neq -1\ , \tag{5}$$

$$H(\bar{S}_p,S_f)\neq 1 \qquad \text{for all } \bar{S}_p\in\bar{U}\ , \tag{6}$$

where the equation (5) means that there is no shadow from any polyhedron existing within the negative-space of S_f. Here, for area sources, there are no shadows when $H(S_e,V)=-1$ because lighting direction is limited.

The penumbra area is obtained as the intersection region of the penumbra volume of V and the face S_f. The points consisting the boundaries of penumbra are easily obtained as the intersection points of the face S_f and the edges between $\hat{S}$ and $\bar{S}$ in Table 1 (e.g., the line including Q_lP_k, Q_lP_k, etc.). The obtained boundaries are transformed to the coordinate system of the image space and are memorized to save memory and to simplify the scanning for the shadow sections on each scan line.

2.5 Illuminance Caluculation

We describe the illuminance calculation for the point having no shadows from any polyhedra and the illuminance calculation in the region shadowed area by one or more polyhedra. Here we assume that the light sources are composed by uniform brightness surfaces, and use the contour integration method for calculating the direct illuminance.

In order to save computation time, the precise illuminance calculation is executed at regular intervals on each scan line, and each pixel in the interval is interpolated in linear except the boundaries of faces and shadows. The illuminance of these parts usually changes abruptly.

(1) Illuminance Calculation of Unshadowed Areas

The illuminance calculation for area sources is as follows: As shown in Fig. 3-a, if the polygon source has m vertices and an intensity of L, the illuminance at a point P on a face S_f is given by the following equation:

$$E=\frac{L}{2\pi}\sum_{l=1}^{m}\beta_l\cdot\cos\delta_l\ , \tag{7}$$

where β_l is the angle between PQ_l and PQ_{l+1}, δ_l is the angle between the face S_f and the triangle P, Q_l, Q_{l+1}.

In the case where the face S_f is the type B(see eq.(3)), the new calculation source is the part of positive side cut by the plane including the S_f, and eq.(7) is applied.

The illuminance calculation for polyhedron sources can be obtained by the following method: In the case where the face S_f is type A, equation (7) is applied for each visible face(front face) of the polyhedron source when viewed from point P, and the illuminance at P is obtained by summing up each illuminance of the visible face. Fig. 3-b shows the contour line of the source when viewed from the point P. As it is clear from the figure, the edges between two visible faces are integrated in the opposite direction from each other and those components of the integration are cancelled. Therefore, the illuminance is obtained by the contour integration method for the contour line of the source. The contour line is defined in a clock-wise direction, but the integration is executed counter-clockwise in order to obtain the positive value.

In the case where the face S_f is type B, the illuminance is obtained by cutting the contour line of the source with the plane including S_f and executing the contour integration of the part of the contour line which exists in the positive-space of S_f.

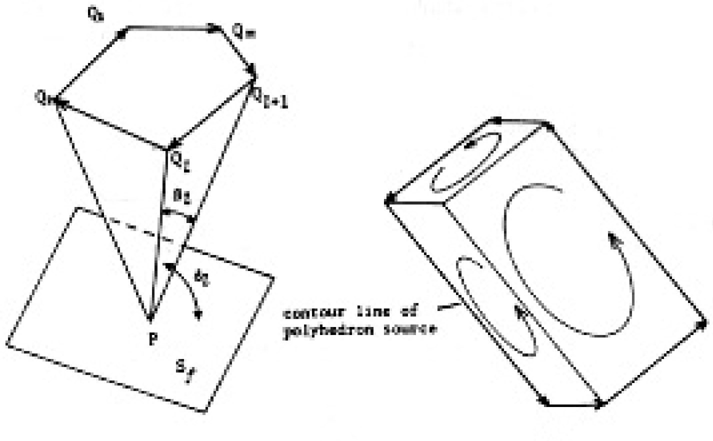

Fig. 3 Illuminance calculation for an area source and an polyhedron source.

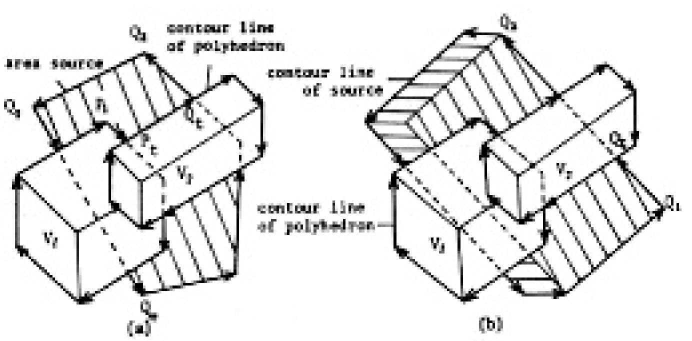

Fig. 4 Illuminance calculation in penumbrae by using the contour integration method.

The umbra area and the penumbra area for a polyhedron source are determined in the same manner of the area source(see Fig.2).

Expanding this idea to three-dimensional space, we consider a shadow volume U_l which is formed by a convex polyhedron V and one vertex Q_l of a source as shown in Fig. 1-a. The shadow volume U_l is the volume(unclosed convex volume) surrounded by the following two types of faces; the planes, each of which has a normal outward pointing vector from U_l, formed by the adjacent vertices $P_{l,i}$, $P_{l,i+1}$ (i=1, 2,.. n_l; n_l+1=1) and Q_l, and the faces of V whose normal faces toward Q_l(front face for Q_l).

Next, we describe the shadow volume caused by an area source S_e and a polyhedron V. A volume which makes an umbra is defined as the common convex volume surrounding U_l(l=1,2,... m); we call this volume an umbra volume. A penumbra volume is defined as the minimum convex volume surrounding U_l(l=1,2,... m), therefore the volume which makes a penumbra is the region inside the penumbra volume and outside the umbra volume.

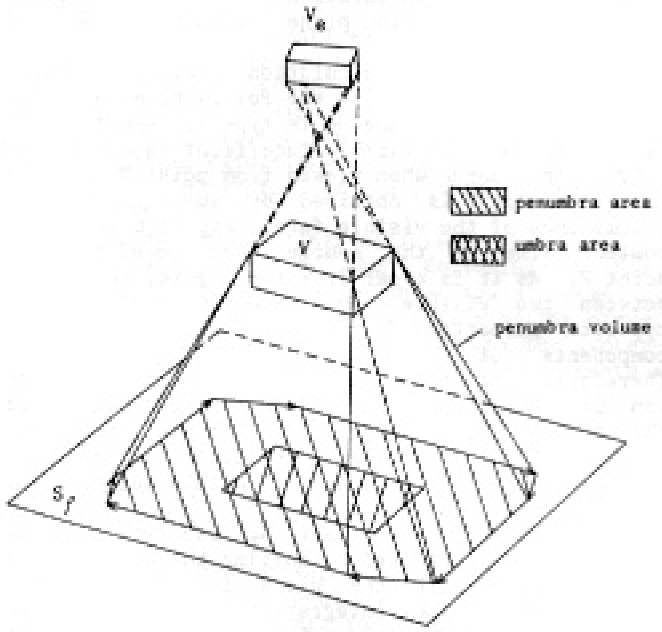

Fig. 2 Regions of umbra and penumbra for a polyhedron source.

In the case where the polyhedron V exists within the negative-space of the source S_e ($H(S_e,V)=-1$), V receives no light, therefore the umbra volume and penumbra volume cannot be defined. Except for this case, the umbra and penumbra volumes are defined as follows:

Assume that $\hat{S}$ is the face formed by a vertex Q_l of a source and an edge of a polyhedron V, $\bar{S}$ is the face formed by a vertex of V and an edge of the source, and $\dot{S}$ is one of the faces of V, then the faces which satisfy Table 1 in the faces of $\hat{S}$, $\bar{S}$ and $\dot{S}$ form the penumbra volume. Here, in the case of $H(S_e,V)=0$, the plane including S_e intersects V, and only a part of V receives the light. Therefore, in this case, the penumbra and umbra volumes are the portion of the positive-space of S_e among shadow volumes obtained by Table 1.

In application of this same manner, the umbra and penumbra volumes for polyhedron sources are formed by the faces satisfying Table 1. However, polyhedron sources have no restriction for their lighting direction unlike area sources, so the penumbra and umbra volumes always exist.

In Table 1, the faces satisfying $H(\hat{S},V)=-1$ are formed by a vertex Q_l of the source and the edges of the contour line of V when viewed from Q_l. Therefore, the faces of the penumbra volume are obtained by the following steps. First, we extract the contour line of V for each vertex of the source, then consider the pyramids formed by the contour and each vertex of source, and finally extract the faces satisfying $H(\hat{S},S_e)=1$(or $H(\hat{S},V_e)=1$) among those pyramids. The faces of the umbra volume are obtained by extracting the faces satisfying $H(\hat{S},S_e)=-1$(or $H(\hat{S},V_e)=-1$) among these pyramids. The faces of $\dot{S}$ satisfying Table 1 are obtained by using the classification of faces(see eq.(3) and(4)). That is, the faces of type A correspond with the penumbra volume, and the faces of type A or B correspond with the umbra volume.

2.4 Shadow Boundaries on Faces

It is inefficient to detect shadows in using the point by point method when illuminance calculation is executed. Here we use the method that calculates the shadow boundaries on each face prior to the illuminance calculation for each point. Namely, the polyhedra casting shadows on each face are extracted. These polyhedra are obtained by the same method

Table 1 Faces constructing penumbra and umbra volumes.

types of face	area source		polyhedron source	
	penumbra volume	umbra volume	penumbra volume	umbra volume
$\hat{S}$	$H(\hat{S},V)=-1$ $H(\hat{S},S_e)=1$	$H(\hat{S},V)=-1$ $H(\hat{S},S_e)=-1$	$H(\hat{S},V)=-1$ $H(\hat{S},V_e)=1$	$H(\hat{S},V)=-1$ $H(\hat{S},V_e)=-1$
$\bar{S}$	$H(\bar{S},V)=-1$ $H(\bar{S},S_e)=1$		$H(\bar{S},V)=-1$ $H(\bar{S},V_e)=1$	
$\dot{S}$	$H(\dot{S},S_e)=1$	$H(\dot{S},S_e)\neq-1$	$H(\dot{S},V_e)=1$	$H(\dot{S},V_e)\neq-1$

$$H(S_f,V) = \begin{cases} 1\ F_{S_f}(P_k) \geq 0 & \text{for all } P_k \in V \\ -1\ :\ F_{S_f}(P_k) \leq 0 & \text{for all } P_k \in V \\ 0\ :\ \text{else} \end{cases} \quad (2)$$

where $H(S_f,V)=1,-1$, and 0 means that V exists within the positive-space of S_f, V exists within the negative-space of S_f, and V intersects with the plane including S_f, respectively.

(3) The shape of every area source is a convex polygon consisting of m vertices, and the illuminating direction of the sources is only toward the positive-space. A polyhedron source is a convex polyhedron with all its faces bright.

(4) The contour line of a polyhedron V viewed from an arbitrary point Q is defined as follows; the string of edges consisting of two types of faces, one incudes Q within the positive-space and the other includes Q within the negative-space; where the string of vertices is defined in a clockwise direction when viewed from the point. The contour line viewed from the point Q always appears to be a convex polygon.

(5) In order to simplify the shadow processing and the illuminance calculation, the faces of polyhedra are classified into the following types corresponding to the positional relationship between each face and the source. For an area source S_e, the face S_f is classified into the following three types by using equation (2):

$$\begin{aligned} &\text{Type A : } H(S_f,S_e)=1, \quad H(S_e,S_f)\neq -1 \\ &\text{Type B : } H(S_f,S_e)=0, \quad H(S_e,S_f)\neq -1 \qquad (3) \\ &\text{Type C : } H(S_f,S_e)=-1 \text{ or } H(S_e,S_f)=-1. \end{aligned}$$

In the same manner, the face S_f is classified into the following three types for a polyhedron source V_e:

$$\begin{aligned} &\text{Type A : } H(S_f,V_e)=1 \\ &\text{Type B : } H(S_f,V_e)=0 \qquad (4) \\ &\text{Type C : } H(S_f,V_e)=-1\ , \end{aligned}$$

where a face of type A receives the light from whole region of the source, a face of type B receives the light from a part of the source, and the face of type C receives no light from the source at all.

Therefore, the illuminance calculation of the face of type C is unnecessary because of shaded face, but for the faces of type A and C, the illuminance calculation taking into account the shadows from other polyhedra is required. The illuminance for the faces of type B can be obtained by setting a new source, a part of the original source occupying the positive side of the plane including S_f.

2.2 The Outline of The Procedure

i) Input object data, viewpoints, view directions, angles of the view fields, and light sources.

ii) Project the vertices of objects onto the perspective plane, and calculate the priority of visibility for the overlapped polyhedra on the perspective plane.

iii) Classify the faces of each polyhedron on the positional relationship between the source and each face(see eq.(3) and (4)), and decide if the face has shadows or receives no light.

iv) Obtain the volumes making umbra and penumbra for each polyhedron.

v) Extract the polyhedra casting shadows on the visible faces, and calculate the shadow boundaries on them.

vi) Scan the perspective plane from top to bottom, that is, remove the hidden surfaces, calculate the shadow sections(umbra and penumbra) of the visible faces on each scan line, and calculate the illuminance for each point on the scan line.

When the viewpoints and/or the view directions change, the steps ii) through vi) are repeated. For movement of the source position, the steps iii) through vi) are executed.

This paper discusses iii), iv), v) and vi). See Ref.[5] for i), ii) and hidden surface removal.

2.3 Shadow Volumes for Area Sources or Polyhedron Sources

In order to simplify the explanation, we explain the shadow boundaries caused by a convex polyhedon and an area source.

We define the contour line C_l of a convex polyhedron V when viewed from a vertex Q_l of an area source, and represent the vertices of C_l with $P_{l,i}$ ($i=1,2,\ldots,n_l$; n_l=the number of vertices of C_l) of V consisting of vertices P_k ($k=1,2,\ldots n$; n=the number of vertices of V). The contour line C_l appears to be a convex polygon when viewed from the vertex Q_l. Here the numbering of $P_{l,i}$ is clockwise.

As shown in Fig. 1, a projected contour line C_l' is defined as the projection of C_l onto the face S_f. Then, the common convex part of the projected contour lines C_l' ($l=1,2,\ldots,m$) makes an umbra area, and the minimum convex polygon surrounding C_l' ($l=1,2,\ldots,m$) with the exception of the umbra area, forms a penumbra area.

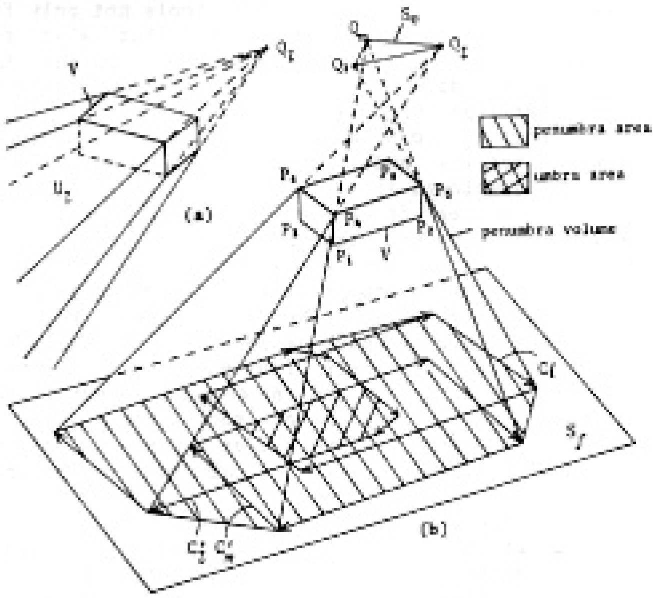

Fig. 1 Regions of umbra and penumbra for an area source.