Modeling of Skylight and Rendering of Outdoor Scenes

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Abstract
Photorealistic animated images are extremely effective for pre-evaluating visual impact of city renewal and construction of tall buildings. In order to generate a photorealistic image not only the direct sunlight but also skylight must be considered.

This paper proposes a method of high-fidelity image generation for photorealistic outdoor scenes based on the following ideas:

1. The intensity distribution of skylight taking account of scattering and absorption due to particles in the atmosphere which coincides with CIE standard skylight luminance functions is sought, and realistic images considering about spectrum distribution of skylight for any altitude of the sun can be easily and accurately displayed.

2. A rectangular parallelepiped with a specialized distribution of intensity simulating the skylight is introduced for efficient calculation of illumination due to skylight, and by employing a graphics hardware calculation of the skylight illumination taking into account shadow effects is obtained with high efficiency; these techniques can be used to generate sequences of images, making animations possible at far lower calculation cost than previous methods.

Keywords: Skylight, Photo-realistic image synthesis, Animation, Visual environmental assessment.

1 Introduction

Computer generated images for visual environmental assessment must satisfy the following requirements: (1) Models must be accurate to true natural light especially on the color of sky, and (2) Generation of animations must be possible to allow the scenes to be evaluated from different directions with high quality images and low cost.

In regard to the former, sky color was calculated based on the change of spectral distribution due to particles in the atmosphere [1]. The method, however, leaves the following problems; it does not discuss about the CIE standard and just shows some examples, the sunlight pass becomes infinitive when the altitude of the sun is low because of the assumption of the atmosphere as flat, and density of air particles is assumed as two constant value layers. A method to obtain spectral distribution at any point in the sky under any weather conditions using the empirical formula obtained through measurement was proposed [2]. In this method, at first the luminance distribution is calculated by CIE standard skylight luminance functions, and luminance is first converted into correlated color temperature, and then into spectral distribution [3].

The purpose of this paper is to construct a model as accurate to actual natural light as possible. Our method is based on the actual physical model in the same manner as Klassen, but we researched and discovered the coefficients which are suitable for the CIE standard for any position of the sun. This makes it easier to use skylight as a light source in Computer Graphics.
In regard to the latter, the authors proposed a shading model which takes into account the skylight as an accurate ambient light source [4], but it does not take account of any spectral distribution of sky, even though it employed CIE standard for the distribution of skylight. And this method gives accurate illumination due to skylight, but the calculation cost is high, especially serious in the process of calculating contours of all objects from every calculation point and detecting intersections of contours and sample lines. Moreover, as this method does not take into account the intensity distribution of the sky when the hemisphere is divided into band sources, the sky dome must be divided into considerably narrow band sources for accurate calculation. For example the luminance distribution of clear sky forms roughly concentric circles; the intensity of skylight near the sun (the center of the circles) is extremely high, the intensities from other directions are considerably lower, and the range of intensity is wide.

In addition, a shading model with specular reflection considering spectral distributions of both direct sunlight and skylight was developed [5], a method for rendering solar penumbra taking into account the size of the sun was proposed [6]. These techniques have enabled great progress to be made in photorealistic representation; they are extremely useful for visual environmental assessment under outdoor conditions.

On the other hand, for lighting simulation of indoor scenes, the radiosity method has been [7, 8], widely recognized for its graphic ability to represent realistic images. Algorithms for fast calculation of radiosity [9, 10, 11] and high accuracy [12, 13] have also been proposed. These techniques suggest us to use the hardware to calculate illumination due to skylight.

In our new method the contributions of skylight intensities are calculated at elements on a movable parallelepiped with a shape suitable for efficient calculation of illumination due to skylight, and the determining of whether or not each element is obscured by obstacles is performed using graphics hardware. The parallelepiped introduced in this paper is similar to the hemi-cube method [7], but its shape and direction vary according to the direction of the sun and the luminance distribution of skylight. For faster calculation graphics hardware should be used, and for accurate calculation of sky illuminance the size of the elements in regions of high intensity distribution should be smaller than in low intensity regions.

2 Scattering Model of Atmosphere

Our method is similar to the method of Klassen [1], but we assume the following conditions in our approach to actual physical phenomena; (1) The atmosphere is considered as a spherical-shell atmosphere. (2) The density distribution of both air molecules and aerosols vary exponentially with altitude. (3) Light from multiple scattering may be ignored. (4) For visible wavelengths, absorption into the ozone layer is negligible compared to that by air molecules and aerosols. (5) Reflection from the ground is ignored. (6) Light travels on a straight line, even though the actual path is curved due to the variation of the index of refraction with altitude.

2.1 Scattering Due to Air Molecules

The light reflected due to air molecules is generally called Rayleigh scattering as is well known. If it is assumed that atmosphere has a spherical shape and that the density of air molecules decreases exponentially with altitude, light intensity $I_r(\lambda)$ reaching view point $P_r$ can be obtained by using the numerical integration from the point outside the atmosphere, $P_o$, to the point $P_r$ as below (see Fig. 1).

$$I_r(\lambda) = I_s(\lambda) k F_r(\theta) \frac{1}{\lambda^4} \int_0^{H_o} \exp(-t(s, \lambda) - t(s', \lambda)) \rho(s) \, ds,$$

$$k = \frac{2 \pi^2 (m^2 - 1)^2}{3 N_s}.$$
\[ F_s(\theta) = \frac{3}{4} (1 + \cos^2 \theta), \]  
\[ \rho = \exp \left( -\frac{h}{H_0} \right), \]  
where \( \lambda \) is the wavelength of incident light; \( m \), the index of refraction of the air; \( N_s \), the molecular number density of standard atmosphere; \( \theta \), the scattering angle. \( t(s, \lambda) \) is the optical length from a point in the atmosphere, \( P \), to \( P_s \); \( t(s', \lambda) \), the optical length from the top of the atmosphere, \( P_0 \), to \( P_s \); \( \rho(h) \), the density ratio at height \( h \); \( H_a \), the distance between \( P_0 \) and \( P_s \). \( H_0 \) is the scale height of the air molecules, which corresponds to the thickness of the atmosphere if the density were uniform. \( m \) is a function of wavelength but about 1.00028 for visible wavelengths. \( N_s \) is \( 2.7 \times 10^{19} \text{ cm}^{-3} \) and called the Loschmidt number. \( H_0 \) is 7994 m, and the thickness of the atmosphere, \( H \), is 30 km. The attenuation coefficient \( \beta \) (i.e., the extinction ratio per unit length) is generally given by

\[ \beta = \frac{4\pi k}{\lambda}. \]  

From equation (1) we know that the light intensity is inversely proportional to \( \lambda^4 \).

2.2 Scattering Due to Aerosols

Aureole is one of the effects of scattering due to aerosols. This is an effect occurring around the sun due to the extremely strong intensity of scattering due to aerosols within a small scattering angle (usually less than 10 degrees).

Scattering by aerosols is described by the Mie scattering theory, and depends on particle size and light wavelength. Because the size range of particles such as dust and moisture in the atmosphere is very large, aerosol scattering cannot be calculated in as straightforward a manner as this due to air molecules. As the Mie scattering theory is very complex, we employ an approximation. Klassen [1]
used a phase function which Blinn introduced in reference [15]. Gibbons proposed phase functions for a foggy atmosphere in reference [16].

\[ F_r(\theta) = a(1 + 9 \cos^{16} \theta) : \text{hazy atmosphere} \]  
\[ F_r(\theta) = a(1 + 50 \cos^{64} \theta) : \text{murky atmosphere} \]

So, we use the phase function for a hazy atmosphere to approximate that due to aerosols.

The attenuation coefficient of aerosols (corresponds to eq. (5)) is expressed by the form \( B \lambda^{-b} \), and the experimental value of \( b \) is generally found to be 1.0. \( B \), the turbidity coefficient, varies with the ratio of aerosols in atmosphere \((B = 0.05 \sim 0.2)\).

Alike the density distribution of air molecules, the density of aerosols decreases exponentially with altitude (up to 10 km); the scale height, \( H_0 \), of eq. (4) is set to 1.2 km [17]. Light scattering only occurs near the sun, but absorption occurs over the whole space. Therefore, the absorption due to aerosols is taken into account for the optical length in eq. (1). As a result optical length increases.

### 2.3 Comparison with CIE standard distribution

Taking into account the effect of both air molecules and aerosols, we compare the luminance distribution obtained from our method with CIE standard luminance distribution. In our experiment, \( a \) is about 0.02, and \( B \) is about 0.09, and result is shown in Fig. 2.* The upper line shows the luminance distribution of CIE standard and the middle line is the result of our method with a sun altitude that varies from 10 to 90 degrees. The values are shown in 50 grade and maximum value is corresponds to the color red. The bottom line shows the difference between CIE standard and our method. These figures show that for each solar altitude our method coincides with CIE standard except for part of the horizontal, and the maximum value remains under 20%.

### 3 Calculation of Illumination due to skylight by Using Parallelepiped

#### 3.1 Previous Methods and Proposed Basic Ideas

In previous shading models considering sky light [4], the sky is considered to be a hemisphere of large radius (the “sky dome”, see Fig. 3) that acts as a diffuse light source with nonuniform intensity. In order to calculate sky illuminance at a calculation point \( P \) taking into account the effect of obstacles to sky light, the sky dome is subdivided into several band sources, and visible parts of the sky are determined along sample lines which coincide with the center line of each band source, and finally sky illuminance is obtained by summing up the intensity of the visible parts. This method gives accurate sky illuminance, but the calculation cost is high, especially serious in the process of calculating contours of all objects from every calculation point and detecting intersections of contours and sample lines. Table 1 shows the calculation time of a scene consisting of 99 polyhedra and rendered with 80 band sources. It is obvious that the bottleneck of the method considering sky light is the calculation costs on contours of objects and intersections of contours and sample lines. As this method does not take into account the intensity distribution of the sky, the sky dome must be divided into considerably narrow band sources for accurate calculation of sky illuminance. For example, as shown in Fig. 2, the intensity distribution of clear sky forms roughly concentric circles; the intensity of sky light near the sun (the center of the circles) is extremely high, the intensities from other directions are considerably lower, and the range of intensity is wide. Furthermore, the solar position (the center of the circles) changes with the passage of time. On the other hand, for overcast sky the difference of intensities over the entire dome is smaller and the range is narrower than for clear skies. In previous methods,

* See page C–521 for Figure 2.
the sky was divided such that the projected areas of each band source onto the surface including the calculation point were uniform; these methods are thus susceptible to sampling error for obstructions, especially in regions with high intensity distribution.

In order to address these problems, in our new method the contributions of sky light intensities are calculated at grid elements on a movable parallelepiped with a shape suitable for efficient calculation of sky illuminance, and the determination of whether or not each grid element is obscured by obstacles is performed using the workstation's graphic hardware.

Table 1: Timing test of the previous method.

<table>
<thead>
<tr>
<th>process</th>
<th>rate of the time (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>detecting intersections between objects and lines</td>
<td>61.5</td>
</tr>
<tr>
<td>calculating contours of objects</td>
<td>12.0</td>
</tr>
<tr>
<td>calculating specular reflection</td>
<td>5.7</td>
</tr>
<tr>
<td>detecting visible areas of the sky</td>
<td>5.0</td>
</tr>
<tr>
<td>calculating sky illuminance</td>
<td>2.9</td>
</tr>
<tr>
<td>the others</td>
<td>12.9</td>
</tr>
</tbody>
</table>

(number of polyhedra: 99, number of band sources: 80)

Figure 3: Sky dome for calculating sky illuminance.

Illumination due to skylight is calculated in the following steps.  
(1) Each face of the parallelepiped is adaptively subdivided into a number of elements, and the contribution of skylight intensity for every element is calculated (see Fig. 4). (2) In order to determine whether or not each element is obscured by objects in the scene, all objects are perspective-transformed onto each surface of the parallelepiped using graphics hardware (see Fig. 5). (3) Sky luminance is calculated by summing up contributions assigned to each element which is not obscured by objects.
Figure 4: A parallelepiped for calculating sky illuminance.

Figure 5: Perspective-transformation onto the surface of parallelepiped

3.2 Calculation of Contribution Coefficient

The skylight intensity contribution coefficient of an element depends upon both the skylight intensity and the solid angle of the element. If it is assumed that the skylight intensity in element $i$ is constant,
\( L_i \), the solid angle, \( dS_i \), of element \( i \) is expressed by the following equation (see Fig. 6).

\[
dS_i = \frac{\Delta S_e \cos \psi}{r^2},
\]

(8)

here \( \Delta S_e \) is the area of element \( i \), \( \psi \), the angle between the normal of element \( i \) and the vector from the calculation point to the center of element \( i \), and \( r \), the distance between the calculation point and the element. Then, the contribution coefficient \( C_i \) of skylight intensity for element \( i \) is expressed by the following equation;

\[
C_i = \frac{L_i dS_i}{2\pi}.
\]

(9)

Figure 6: Solid angle of an element.

### 3.3 Setting a Parallelepiped

In this section, we show how to create the parallelepiped taking into account the intensity distribution of a clear or overcast sky. Fig. 2 shows that the intensity distribution of a clear sky varies in radical with the position of the sun; On the other hand, the intensity distribution of an overcast sky is not depend upon the solar altitude; skylight intensity is always highest near the zenith.

It is considered that when contribution of every element are alike illumination due to skylight can be calculated accurately, so the parallelepiped is set in the following manner (see Fig. 4): (1) The top surface should always face the direction of the highest intensity. (2) The element interval of the top surface should be smaller than that of the side surfaces. (3) The angle \( \varphi \) which specifies the size of the top surface should be appropriately determined taking into considering of the intensity distribution of the sky.

For processing steps, (2) and (3), the most effective and accurate calculation can be expected when the contributions are approximately uniform for every element.

### 3.4 Optimizing the Parallelepiped

In order to optimize the size of the top of the parallelepiped and the fineness of its element, the ratio of the fineness of the top and side faces, \( m \), and the angle of the top surface, \( \varphi \), are separately determined.
Taking into account the considerable difference in intensity distributions of clear and overcast skies, parameters $m$ and $\varphi$ which minimize the following function $g$ are determined.

$$g(m, \varphi) = \frac{C_{\text{max}}}{C_{\text{min}}}$$

where $C_{\text{max}}$ and $C_{\text{min}}$ are the maximum and the minimum contributions for all elements on the parallelepiped determined by $m$ and $\varphi$.

Because of the difficulty of analytical calculation, we use an iterative method to determine the optimal values for parameters $m$ and $\varphi$. Fig. 7 shows the relationship between the estimation function and the parameters for the case of an overcast sky; $m$ and $\varphi$ which minimize $g$ are roughly 1.15 and $38^\circ$, and the value of $g$ for these parameters is about 3. These values are used for determining the appropriate parallelepiped for an overcast sky. Fig. 8 shows the result of iteration for determine optimal parameters for a clear sky. A lookup table of optimal values for parameters $m$ and $\varphi$ as a function of the solar altitude can be generated by using the result.

**Figure 7:** Relationship between ratio of fineness $m$, angle $\varphi$, and estimation function $g$.

**Figure 8:** Optimal $m$ and $\varphi$ for a clear sky.

### 3.5 Calculation of Illumination Due to Skylight

In order to calculate sky illuminance taking into account the effect of obstructions, a graphics hardware is used to perspective transform of all objects, from the point of view of each calculation point onto each surface of the parallelepiped. The elements of the parallelepiped correspond to the pixels after
perspective transforming the object onto the surfaces, here only perspective transformation and scan conversion must be executed.

Sky illuminance at calculation point P is obtained by the following equation.

$$ I = \sum_{i=1}^{n} H_i C_i \cos \xi_i, $$

where $H_i$ is the following function:

$$ H_i = \begin{cases} 
0 & \text{element i obscured} \\
1 & \text{element i not obscured} 
\end{cases} $$

Value of the function $H_i$ can be obtained using the pixel value after perspective-transformation. $n$ is the total number of elements, and $\xi_i$ is the angle between the normal of a surface illuminated by skylight and the vector from calculation point P to element i (see Fig. 6).

3.6 Comparison of the Parallelepiped Method with Band Source Method

In order to compare the parallelepiped method with the band source method, we show variety of RMS when the number of both bands and elements in Fig. 9. The solar altitude is 20 degree. When the number of bands is 40 and the number of elements on the top surface is 40×40, the value of RMS becomes 1.5 as shown in Fig. 9. It is shown in Fig. 10 that the variety of calculating time as the number of both bands and elements varies. As shown in Fig. 10, the greater the number of objects, the more the application of our method becomes favorable.

![Figure 9: Variety of RMS.](image1)

**Figure 9: Variety of RMS.**

![Figure 10: Calculation time.](image2)

**Figure 10: Calculation time.**

4 Rendering Images Using Illuminance Mapping

This section describes a method of rendering images using illuminance mapping; sky illuminance is calculated in advance at every sampled point on each surface, and when rendering images, sky
illuminance at an arbitrary point is interpolated by using those sampled points. The following processes are executed once as a pre-processing for one scene (see Fig. 11).

1. Each object surface is divided into quadrilaterals. (Triangles are also treated as a quadrilateral with one doubled vertex.)

2. Each quadrilateral is subdivided into small patches until the lengths of its edges become shorter than a threshold value set in advance.

3. Sky illuminance is calculated at all four vertices of each patch using the method described in the previous section, and stored in a file.

To render images, the following well-known method is employed for determining the patch containing the sky illuminance calculation point. That is, the \((u, v)\) coordinates of the calculation point are calculated \([18]\) in the \(u-v\) coordinate system determined by two edges of the quadrilateral as shown in Fig. 12. If the number of patch subdivision along the \(u\) and \(v\) axes are \(n_u\) and \(n_v\), then the address of the patch containing the calculation point can be expressed as \([u \cdot n_u] + 1, [v \cdot n_v] + 1\) (the notation \([x]\) means that the fractional portion of \(x\) is discarded). Similarly, considering the \(u'\) and \(v'\) coordinate system along two edge of a patch as shown in Fig. 12, the coordinates \((u', v')\) of the calculation point are expressed by the following equations.

\[
\begin{align*}
  u' &= u \cdot n_u - [u \cdot n_u], \\
  v' &= v \cdot n_v - [v \cdot n_v].
\end{align*}
\]  

Sky illuminance of the calculation point is interpolated using the \((u', v')\) coordinates and sky illuminances at the four vertices of the patch, \(I_1, I_2, I_3\) and \(I_4\):

\[
I_p = (1 - u')(1 - v')I_1 + (1 - u')v'I_2 + u'v'I_3 + u'(1 - v')I_4.
\]  

The illuminance at each face consists of two elements, the illuminance caused by skylight and the direct sunlight. In practical measurement, however, the both of them on a horizontal plane are observed together. Then separating techniques of those elements have been developed by many pioneers \([19], [20], [21]\).

![Figure 11: Pre-process of rendering images.](image1)

![Figure 12: Coordinate systems on a surface.](image2)
5 Examples

The proposed method has been applied to a city renewal planning. Fig. 13 (a) shows a scene under an overcast sky and a clear sky. Fig. 13 (b) shows a scene under a clear sky. The solar altitude in Figs. (b) is 5°. Fig. 13 (c) illustrates another scene under a clear sky. The solar altitude is the same as that in Fig. 13 (b).

The proposed method uses mapping of illuminance as well known in radiosity method. The calculation time of the proposed method was measured on Fig. 13; the numbers of polyhedra and the elements on the parallelepiped are approximately 800 and 3200, respectively. The pre-processing time (calculating illuminance) was about twelve hours, and the image generation time for one frame of an animation was roughly one hour by using Silicon Graphics IRIS-4D/120GTX. This image generation time includes the time taken to represent the trees using a technique of imaginary transparent planes [6], the shadow of sunlight, the specular reflection caused by direct sunlight, and the solar penumbral cast by trees.

On the other hand, in the previous method using band sources, calculation time was measured for the subset of the data in Fig. 13 because of extensive calculation time; the number of polyhedra was about 150, and the sky dome was divided into 48 band sources (sky luminance is calculated along only sample lines which coincide with the center line of each band source). It took about 19 hours to generate one image: The proposed method is able to make animation possible at far lower calculation cost.

6 Conclusions

The paper discussed fast image generation of outdoor scenes, taking into account sky light effects, and the method has the following advantages:

(1) The intensity distribution of skylight which very nearly coincides with CIE standard skylight luminance functions are sought by taking account of scattering and absorption due to particles in the atmosphere.

(2) Sky illuminance can be calculated efficiently by employing an imaginary parallelepiped generated for taking into account the intensity distribution of sky light, and a graphic hardware is allowed to be used for faster calculation of the effect of obstacles to sky light.

Images with high quality can be practically used for visual environmental assessment, because the proposed method breaks the bottleneck of sky light calculation cost. Furthermore, based on the proposed method, we can expect to start the development of more realistic outdoor simulations; e.g., a method for fast calculation of specular reflection caused by not only direct sunlight, like in this paper, but also sky light and a method of illuminance mapping with hardware assistance will create more realistic images at a low cost.

References


* See page C-522 for Figure 13.


K. Tadamura et al.: Figure 2. Distributions of luminance of skylight.

CIE standard (upper), proposed method (middle), and errors (bottom).

(from 10° (left one) to 90° (right one) every 10°)
K. Tadamura et al.: Figure 13. Example of city renewal planning.