Skylight for Interior Lighting Design

Yoshinori Dobashi, Kazufumi Kaneda, Takanobu Nakashima, Hideo Yamashita

Hiroshima University
1-4-1 Kagamiyama,
Higashi-hiroshima, 724 Japan

Tomoyuki Nishita
Kastumi Tadamura

Fukuyama University
985 Sanzo, Higashimura-cho,
Fukuyama, 729-02 Japan

Hiroshima Prefectural University
562 Nanatsuka,
Shoubara, 727 Japan

Abstract

It is inevitable for indoor lighting design to render a room lit by natural light, especially for an atelier or an indoor pool where there are many windows. This paper proposes a method for calculating the illuminance due to natural light, i.e. direct sunlight and skylight, passing through transparent planes such as window glass. The proposed method makes it possible to efficiently calculate such illuminance accurately, because it takes into account both non-uniform luminous intensity distribution of skylight and the distribution of transparency of glass according to incident angles of light. Several examples including the lighting design in an indoor pool, are shown to demonstrate the usefulness of the proposed method.

Keywords and Phrases: Skylight, Transparency, Illuminance, Lighting Design, Realistic Image, Natural Light, Luminous Intensity Distribution.

1. Introduction

Computer Graphics is an excellent tool for both lighting design in a room and architectural design in the open air, and has come into wide use. When designing a building, natural light sources, i.e. direct sunlight and skylight, play important roles in synthesizing a realistic image. A lighting model for skylight was first developed in 1986 [1]. The method makes it possible to generate photorealistic images of outdoor scenes.

This model has been improved to render more realistic outdoor scenery. A method for rendering a surface with specular reflection lit by skylight was developed in 1991 [2]. Taking into account spectra of skylight measured under different weather conditions, realistic images of automobiles lit by natural light were rendered [3]. A method for decreasing skylight computation time using graphics hardware was proposed [4]. The method is also able to accurately calculate illuminance due to skylight because a parallelepiped used for illuminance calculation changes in shape and direction according to luminous intensity distribution of skylight. Taking into account the solid angle of the sun, realistic solar penumbras were rendered [5]. These methods have contributed to realistic image synthesis for outdoor scenes.

Skylight is also an important element for lighting design in a room, especially an atelier, a lounge, or a gymnasium where there are many windows and/or skylights. Even in a museum, where there are few windows in order to help prevent works from becoming faded, designers attempt to make use of skylight these days. To calculate accurate illuminance due to skylight passing through windowpanes for indoor lighting design, both of the following factors should be taken into account:

1) non-uniform luminous intensity distribution of skylight.
2) distribution of the transparency of glass according to incident angles of light.

Most methods for lighting design in a room have been developed for artificial light sources. Several methods based on radiosity are able to calculate luminance due to light passing through a translucent plane.
[6, 7, 8]. However, these methods are not suitable for skylight, because they assume light sources with uniform luminous intensity distributions.

The paper proposes a method for lighting design in a room with skylight passing through windowpanes. The method makes it possible to calculate accurate illuminance due to skylight taking into account the two factors described above. For the first factor, we improve the method of dividing the sky dome into band sources. According to the contribution of skylight to each illuminance calculation point, the sky dome is adaptively divided into band sources with different width. To reduce sample error in calculating visible parts from each calculation point, sub-sample lines are introduced in the proposed method.

For the second factor, illuminance due to skylight passing through glass is calculated by adaptively sampling the incident angle of the light, because the transparency of glass varies according to this angle. The proposed method is able to efficiently calculate accurate illuminances by sampling the incident angle until the transparency is almost constant.

In the following sections, we first discuss a method of calculating illuminance due to the direct sunlight passing through glass, and propose an improved method of calculating skylight and a new method of calculating illuminance due to the skylight passing through glass. Several examples including an indoor pool demonstrate the usefulness of the proposed method.

2. Calculation of Illuminance due to Direct Sunlight through Transparent Surfaces

The illuminance on an object lit by natural light consisting of direct sunlight and skylight is calculated by summing the illuminances due to both sources. The illuminance $E_{\text{directlight}}$ due to the direct sunlight passing through transparent surfaces is calculated by the following equation (see Fig. 1).

$$E_{\text{directlight}} = I_\text{dn} \cos(\theta) \prod_{i=1}^{n} \tau_i(\gamma_i),$$

(1)

where $I_\text{dn}$ is the intensity of direct sunlight, $n$ is the number of transparent surfaces between the sun and the calculation point, $\theta$ is the angle between the incident direction and the surface normal at the calculation point, and $\tau_i$ is the transmittance function of the $i$-th transparent surface that depends on the incident angle $\gamma_i$ to the transparent surface. As an example, $\tau_i$ is defined by Eq. 15 described in section 4.1.

The intensity of direct sunlight is obtained by using Bouguer's equation that expresses the illuminance due to the direct sunlight on a surface whose normal is oriented to the sun position [9].

$$I_\text{dn} = I_\text{0} P_{\text{thin}}(\theta),$$

(2)

![Figure 1: Direct sunlight passing through a transparent surface.](image-url)
where $h$ is the altitude of the sun, $I_0$ is the intensity of the sunlight out of the atmosphere, and $P$ is transmittance of the atmosphere. In this paper, we use 0.66 for the transmittance of the atmosphere and 1,327 [W/m²] for the intensity of the sunlight out of the atmosphere [9].

3. Calculation of Illuminance due to Skylight

3.1. Illuminance calculation using band sources

Since the sky is considered to be a hemispherical light source of large radius, we can consider the calculation point as always being at the center of this hemisphere. The sky dome is divided into several band sources (see Fig. 2). The luminance of the band sources is assumed to be continuously varying with respect to longitudinal direction and constant to latitudinal direction of the band source. The central line of the band source is called a "sample line", and a plane including the sample line and the x-axis is called a "sample plane".

Let us assume the tilt angle of the sample plane of the $\ell$-th band source to be $\delta_\ell$, and the direction of sky element $P_e$ to be $(\alpha, \delta)$ where $\alpha$ is the angle from the x-axis (PP₀) to the sky element and $\delta$ is an angle from the horizontal plane to the sky element. The illuminance $E_\ell(\alpha)$ at the calculation point due to the sub-area between angles $0$ and $\alpha$ along the sample line of the $\ell$-th band source is given by

$$E_\ell(\alpha) = \int_{\delta_\ell - \Delta_\ell}^{\delta_\ell + \Delta_\ell} \int_0^\alpha L(\alpha, \delta) \sin \delta \sin^2 \alpha d\alpha d\delta,$$

where $2\Delta_\ell$ is the width of the $\ell$-th band source and $L(\alpha, \delta)$ is the luminous intensity of the sky element. Since it is assumed that the luminous intensity of the band source in the small width, $\delta_\ell - \Delta_\ell < \delta < \delta_\ell + \Delta_\ell$, is constant, Eq. 3 becomes

$$E_\ell(\alpha) = d_\ell \int_0^\alpha L(\alpha, \delta_\ell) \sin^2 \alpha d\alpha,$$

where

$$d_\ell = \cos(\delta_\ell - \Delta_\ell) - \cos(\delta_\ell - \Delta_\ell).$$
To consider the effect of obstacles, visible parts of the sky dome from the calculation point are determined by calculating intersections between each sample plane and objects in the space and extracting the visible sections of the sample line. More details are described in [1].

3.2. Adaptive division of the sky dome

In the previous method [1], the sky dome is divided into a fixed number of band sources that have the same area projected onto the surface at the calculation point. That is, if the number of band sources is specified to \( N \), then the sky dome is just divided into \( N \) band sources so that \( d_\delta \) in Eq. 4 is \( 1/N \). Using this method, the sky dome must be divided into considerably narrow band sources for accurate calculation of illuminance because the method does not take into account the luminous intensity distribution of skylight. This results in increasing the calculation cost.

The luminous intensity distribution of skylight greatly depends on the position of the sun and is far from uniform. For example, the luminous intensity of skylight around the sun is more than three times greater than that of the darkest part on a clear day. The proposed method takes this into account. That is, the sky dome is divided into band sources with almost the same illuminance contributions to the calculation point.

First, the coordinate system is rotated so that the direction of the sun is included in the y-z plane. Let us consider the sample line for dividing the sky dome, that is the intersection of the y-z plane and the sky dome, and the band source with \( 2\Delta_{\text{div}} \) width that includes the sample line (see Fig. 3). We call the band source a division band source in the following discussion.

From Eq. 3, the illuminance \( E_{\text{div}} \) at the calculation point due to the division band source between the angles \( \alpha_f \) and \( \alpha_{f+1} \) is expressed by

\[
E_{\text{div}} = \int_{\frac{\pi}{2} - \Delta_{\text{div}}}^{\frac{\pi}{2} + \Delta_{\text{div}}} \int_{\alpha_f}^{\alpha_{f+1}} L(\alpha, \delta) \sin \delta \sin^2 \alpha \ d\alpha \ d\delta. \tag{6}
\]

Assuming that the luminous intensity of the division band source in the small width, \( \pi/2 - \Delta_{\text{div}} < \delta < \pi/2 + \Delta_{\text{div}} \), is constant, the elevation angle for division of a band source is determined by satisfying the following equation.

\[
\int_{\alpha_f}^{\alpha_{f+1}} f(\alpha) \ d\alpha < \frac{E_{\text{thresh}}}{d_{\text{div}}}, \tag{7}
\]

where

\[
f(\alpha) = L(\alpha, \frac{\pi}{2}) \sin^2 \alpha, \tag{8}
\]

\[
d_{\text{div}} = \cos \left( \frac{\pi}{2} - \frac{\Delta_{\text{div}}}{2} \right) - \cos \left( \frac{\pi}{2} + \frac{\Delta_{\text{div}}}{2} \right). \tag{9}
\]

Here, \( E_{\text{thresh}} \) is a threshold for dividing the sky dome into band sources, and it implies the illuminance at the calculation point due to the division band source between the angles \( \alpha_f \) and \( \alpha_{f+1} \). Using Eq. 7, the elevation angles for division of band sources are determined by the following process.

First, the threshold value, \( E_{\text{thresh}} \), and an angle for sampling the division band source, \( \Delta \alpha_{\text{sample}} \), are specified. To prevent a band source from having a large width, a threshold angle, \( \alpha_{\text{thresh}} \), is also specified. Using the trapezoidal integral, the illuminance due to a small span of the division band source, \( ([k-1]\Delta \alpha_{\text{sample}}, k\Delta \alpha_{\text{sample}}] \) \( (k = 1, 2, ..., \pi/\Delta \alpha_{\text{sample}}) \), is given by

\[
\Delta E_k = \int_{(k-1)\Delta \alpha_{\text{sample}}}^{k\Delta \alpha_{\text{sample}}} f(\alpha) \ d\alpha
\]

\[
= \frac{1}{2} \Delta \alpha_{\text{sample}} \left( f(k\Delta \alpha_{\text{sample}}) + f((k-1)\Delta \alpha_{\text{sample}}) \right) \tag{10}
\]
Using Eq. 10, we find each $p_k$ that satisfies the following equations.

\[
\sum_{k=1}^{p_1} \Delta E_k < \frac{E_{\text{thresh}}}{d_{\text{div}}},
\sum_{k=p_1+1}^{p_2} \Delta E_k < \frac{E_{\text{thresh}}}{d_{\text{div}}},
\vdots
\sum_{k=p_{n-1}+1}^{p_n} \Delta E_k < \frac{E_{\text{thresh}}}{d_{\text{div}}},
\]

where

\[
P_k \Delta \alpha_{\text{sample}} - p_k \Delta \alpha_{\text{sample}} < \Delta \alpha_{\text{thresh}}.
\]

Finally, the angle for division of a band source, $\alpha_q$, is determined by using $p_q$.

\[
\alpha_q = p_q \Delta \alpha_{\text{sample}}.
\]

This method makes it possible to adaptively divide the sky dome into band sources whose illuminance contributions to the calculation point are almost the same.

Suppose $f(\alpha)$ in Eq. 8 is the function shown in Fig. 4. The proposed method divides the sky dome so that each area of $A_1$, $A_2$, ..., $A_n$ is as near equal as possible. In other words, the sky dome is divided into proportionally thinner band sources as the contribution of illuminance due to skylight is larger. Therefore, using the proposed method, the contribution of each band source is almost the same.

**3.3. Determining visible parts of sky using sub-sample lines**

Compared to previous methods, more accurate illuminance due to skylight is calculated by using the proposed method described in the previous section. However, the problem of failing to detect obstacles still remains, because visible parts of the sky are determined by sampling only on sample lines of band sources. This is a kind of aliasing problem. To overcome it, we propose a method of calculating visible parts of the sky by introducing adaptive sub-sample lines.

Figure 5 shows the sky dome viewed from the calculation point. Visible parts of the band source are determined by calculating the intersections between the sample plane and objects and then extracting visible
Figure 5: Aliasing problem in visible part determination.

```
Procedure CalSkylight()
{
  prevVis = sum of the visible sections of the first band source;
  prevI1l = illuminance due to the first band source;
  prevPos = position of the sample line of the first band source;
  For(i = 2; i <= number of band sources; i++) {
    currVis = sum of the visible sections of the i-th band source;
    currI1l = illuminance due to the i-th band source;
    currPos = position of the sample line of the i-th band source;
    call SubSample(prevVis, prevI1l, prevPos,
                   currVis, currI1l, currPos);
  }
}

Procedure SubSample(prevVis, prevI1l, prevPos,
                     currVis, currI1l, currPos) {
  if(|currVis - prevVis| > eps1 & |currI1l - prevI1l| > eps2) {
    generate a sub-sample line at the center
    between prevPos and currPos;
    subPos = position of the sub-sample line;
    subVis = sum of the visible sections of the sub-sample line;
    subI1l = illuminance due to the sub-sample line;
    call SubSample(prevVis, prevI1l, prevPos, subVis, subI1l, subPos);
    call SubSample(currVis, currI1l, currPos, subVis, subI1l, subPos);
  }
}
```

Figure 6: Pseudo code for calculating visible areas using sub-sample lines.

sections of the sample line. The band source is also considered invisible in the sections where its sample line is invisible. This results in an erroneous area, as shown in Fig. 5, and causes visual artifacts in the resulting image (see Fig. 10 (a)). If the sky dome is divided into a sufficient number of band sources, this problem could be ignored. However, it forces up the computation cost.

To address this problem, sub-sample lines are adaptively inserted between the sample lines only where there is the possibility of sampled error. That is, sub-sample lines are generated where both of the following conditions are satisfied.

1. The difference in the sum of the visible sections between adjacent sample lines is greater than the threshold $\eta_1$.
2. The difference in the illuminance between adjacent band sources is greater than the threshold $\eta_2$.  

The first condition means that the visible sections change drastically. If the first condition is satisfied, sampled error may occur. If the second condition is also satisfied, then a sub-sample line is inserted. If the first condition is satisfied and second is not, it is considered that the error is small enough to ignore and a sub-sample line is not inserted. Furthermore, a sub-sample line is recurrently inserted between the sub-sample lines. It is possible to control the accuracy of skylight illuminance by changing the maximum depth of recursion and the thresholds for the two conditions described above. Figure 6 shows a pseudo code for calculating visible parts using sub-sample lines.

4. Calculation of Illuminance due to Skylight through Transparent Surfaces

In this section, we discuss a method for calculating the skylight illuminance passing through transparent surfaces. In this paper we assume that the refraction effects are ignored because we are interested in thin transparent surfaces such as window glass. For simplicity, we first discuss skylight passing through only a single transparent surface, and then an extended algorithm to handle two or more such transparent surfaces is described.

Figure 7: Calculation of illuminance due to skylight through a transparent surface.

4.1. Illuminance due to skylight through a transparent surface

Here we discuss skylight passing through a single transparent surface. Figure 7 shows a transparent surface intersecting with a sample plane of the \( \ell \)-th band source. If there is no transparent surface, the illuminance at the calculation point \( P \) due to the \( \ell \)-th band source is given by Eq. 4. Suppose that the section between \( \alpha_1 \) and \( \alpha_2 \) of the \( \ell \)-th band source is visible through the transparent surface as shown in Fig. 7. The illuminance due to the \( \ell \)-th band source through the transparent surface, \( E_{\text{glass}, \ell} \), is given by

\[
E_{\text{glass}, \ell} = d \int_{\alpha_1}^{\alpha_2} L(\alpha, \beta) \sin^2 \alpha \, d\alpha,
\]

where \( \tau(\gamma) \) is the transmittance of the transparent surface and \( \gamma \) is the incident angle of skylight to the transparent surface. As an example, the transmittance of a window glass is calculated by the following experimental equation [10].

\[
\tau(\gamma) = 3.4167\cos\gamma - 4.3890\cos2\gamma + 2.4948\cos3\gamma - 0.5220\cos4\gamma.
\]
Figure 8: Discretizing the transmittance of a window glass.

Figure 8 shows the transmittance expressed by Eq. 15. In the proposed method, the transmittance is discretized at the incident angles $\gamma_0, \gamma_1, ..., \gamma_n$, where the transmittance is assumed to be constant between the discretized incident angles. The discretized transmittance is shown in Fig. 8 as dotted lines. Here,

$$\eta(\gamma) = \begin{cases} \tau_1 (0 < \gamma < \gamma_1) \\ \tau_2 (\gamma_1 < \gamma < \gamma_2) \\ \vdots \\ \tau_n (\gamma_{n-1} < \gamma < \frac{\pi}{2}) \end{cases}$$

(16)

$$\tau_s = \frac{\tau_k + \tau_{k-1}}{2}.$$  

(17)

The transparent surface is divided into several areas where the transmittance is constant as shown in Fig. 7. Thus, Eq. 14 is transformed by

$$E_{\text{glass},k} = d_{\text{glass}} \left[ \eta(\gamma_1) \int_{\alpha_1}^{\alpha_2} g(\alpha) \, d\alpha + \eta(\gamma_2) \int_{\beta_1}^{\beta_2} g(\alpha) \, d\alpha + \cdots + \eta(\gamma_n) \int_{\beta_{n-1}}^{\beta_n} g(\alpha) \, d\alpha \right].$$

(18)

$$g(\alpha) = L(\alpha, \delta) \sin^2 \alpha.$$  

(19)

Here, $\eta(\gamma_k)$ is the transmittance corresponding to each section (see Fig. 7), and $\beta_k$ is the intersection of the sample plane and the cone whose apex is located at the calculation point $P$, spread angle is $\gamma_k$, and the axis is the perpendicular from the calculation point to the transparent surface. By using Eq. 18 for each band source, the illuminance due to skylight passing through a transparent surface is calculated.

4.2. Algorithm for multiple transparent surfaces

Based on the method described in the previous section, we propose an algorithm that calculates the illuminance due to skylight passing through multiple transparent surfaces.
First, using the method for a single transparent plane, constant transmittance sections of each transparent surface are calculated. Figure 9 (a) shows a simple example where two transparent surfaces exist, and Figures 9 (b) through (d) show how to extract the constant transmittance sections.

1. For each transparent surface, calculate visible sections of the sample line through the transparent surface (see Fig. 9 (b)).

2. For each transparent surface, using the method for a single transparent plane, calculate the sections where transmittance is constant (see Fig. 9 (c)).

3. All sections of all transparent surfaces are merged into and sorted in the angle order (see Fig. 9 (d)).

After these processes, the transmittance of each section is considered to be constant. For example, in Fig. 9 (d), the transmittance of the section $s_1$ is $\tau_2$, and the transmittance of the section $s_3$, where both the transparent surfaces A and B overlap, is $\tau_2 \times \tau_3$. Thus, multiplying the illuminance due to skylight for each section by the corresponding transmittance, the skylight illuminance passing through multiple transparent surfaces is obtained efficiently.

5. Examples

To investigate the accuracy of the proposed method, we applied the new technique to a simple model of a room whose surfaces have only Lambertian reflection. The first model we used is a room with a glass-less window, and the luminous intensity distribution of the sky is assumed to be CIE standard distribution for clear sky [11].

Figure 10 (a) shows a rendered image and luminance distribution calculated by using a traditional method, a method of dividing the sky dome into a fixed number of band sources. The sky dome is divided into 40 band sources. The discontinuous brightness on the wall spoils the image. Figure 10 (b) shows the results of the proposed method. For adaptive division of the sky dome, we set the parameters $\Delta \alpha_{\text{sample}}$ and $\alpha_{\text{thresh}}$ to 0.5 and 9.0 degrees, respectively, and for adaptive sub-division of band sources, the threshold values of
visible areas, \( \varepsilon_1 \), and illuminances, \( \varepsilon_2 \), were set to 0.1 radians and 10 luxes. We also rendered an image with almost the same quality as Fig. 10 (b) by using the traditional method. The image is shown in Fig. 10 (c) where the sky dome is divided into 360 band sources. The tests were run on a SiliconGraphics IRIS Indigo (85 MIPS). The computation time is shown in Table 1. These results shows the usefulness of the proposed method.

Figure 11 (a) shows an image of a room with a windowpane rendered by the proposed method and its luminance distribution. Compared with Fig. 10 (b), the room becomes darker because of the attenuation of natural light due to the window glass. The brightness of the area around the window decreases more than that of the walls because the transparency of the glass proportionally decreases, the larger the incident angle of skylight is. Taking into account interreflection of light between the surfaces, the image becomes more realistic (see Fig. 11 (b)).

Finally, the proposed method was applied to designing an indoor pool, and the resulting images are shown in Fig. 12.* To express the effect of colored glass, the transmittances for each red, green, and blue component are multiplied by different coefficients. In these figures, we used blue glass. Transmittances for red and green are half times that of blue. To simulate the effect of natural light at two different times of day, the spectra of both direct sunlight and skylight are also taken into account [4, 12].

6. Conclusions

This paper has proposed a method for calculating the accurate illuminance due to sky light. In order to take into account non-uniform luminous intensity distribution of skylight, the sky dome was adaptively divided into band sources whose illuminance contribution to the calculation point is almost the same. To address the sampled error of detecting obstacles, a sub-sample line is recurrently inserted.

We have also proposed a method for calculating the illuminance due to skylight passing through multiple transparent planes. Taking into account the distribution of the transparency of glass according to incident angles of light, the proposed method is able to render realistic images.

Enhancing the proposed method to handle surfaces with specular reflection could also make it more useful.

References


* See pages C-522 and C-523 for Figure 12.


(a) a room with a windowpane.

(b) Taking into account interreflection of light between the surfaces.

Figure 11: Rendered image and luminance distribution of a room lit by natural light.
(a) Traditional method with 40 band sources.

(b) The Proposed method.

(c) Traditional method with 360 band sources.

Figure 10: Rendered image and luminance distribution of a room lit by natural light.