Visual Simulation of Solar Photosphere Based on Magnetohydrodynamics and Quantum Theory

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Summary We propose an efficient method based on physical laws to model phenomena near the surface of the sun, which is known as the solar photosphere. In the field of astronomy, some physical models have been developed to simulate the sun’s turbulence. Most of the previous models are difficult to be applied in the making of CG images since they require a considerable amount of time even when using a supercomputer. The subject of our research is the visual simulation of the phenomena observed above the photosphere, namely solar prominences, because these are characteristic phenomena of the sun that can greatly influence the visual impact in movies and games. The sun is mainly composed of ionized plasma, and its behavior can be treated as a fluid. However, unlike gas, the plasma fluid is influenced by the magnetic field. Thus, magnetic field calculations are needed to calculate the plasma behavior. We use the magnetohydrodynamics (MHD) equations to simulate the behavior of plasma. We propose a new method which can simulate a prominence in a practical computation time. The computation cost is reduced by simplifying the phenomena inside the sun: we only consider the phenomena after the solar prominence erupts because the phenomena before the eruption does not manifest itself in a visual way. To render the simulation results, we emulate an observation method that extracts the specific spectrum emission from the solar plasma.

Key words: Solar phenomenon, Plasma simulation, Magnetohydrodynamics

1. Introduction

Visualization methods of planets, for example Jupiter1) and the Earth23), have been researched, and are being used to create images for commercial films, movies, and computer games. However, visual simulation method for the sun has not been developed in the field of CG. Among the various phenomena associated with the sun (see Fig. 1), we simulate solar prominences because they are observable and distinguishable features above the sun’s surface.

In a conductor such as a plasma fluid, the magnetic field changes and electric current is generated, due to the fluid flow. Consequently, interaction between the electric current and the magnetic field exerts an external force on the plasma fluid. Such complex behavior...
is described by the MHD equations.

Although it is possible to simulate the entire plasma fluid inside and outside the sun with the complex behavior in consideration, its computation cost is too expensive for an application in the CG field even when using a supercomputer. In this paper, we propose a new simulation model for the solar prominence subjected to practical applications in the CG field. The computation cost is reduced mainly by simplifying the phenomena inside the sun: we do not try to simulate the trigger of the solar prominence but simulate the solar prominence after it is triggered. We believe that this simplification is acceptable for applications in the CG field because the phenomena inside the sun cannot be observed. We focus on the physically based simulation above the sun, and use magnetic field lines and particles to represent the fine structure of the solar prominence, which is difficult to be captured using a coarse grid.

For rendering, we calculate the specific wavelength that the plasma emits and as used in astronomy, its intensity is converted to RGB colors using a pseudocolor.

2. Related Work

Fluid simulation is one of the important research fields in CG. Stam proposed a stable technique for simulating fluid motion even when the timestep is large by using a semi-Lagrangian advection scheme to calculate the advection term in the Navier-Stokes equations\(^4\). Fedkiw et al. introduced a technique called vorticity confinement to model small scale vortices that cannot be represented by the simulation with coarse grids\(^5\). Not only the behavior of the fluid but also the interaction of the plasma fluid and the magnetic field is important in the sun. Though the technique for simulating the magnetic field was proposed by Thomaszewski et al.\(^6\), only the magnetism of rigid bodies is calculated as an influence of magnetic fields. The research to calculate the interaction of the magnetic field to the fluid has not been developed in the field of CG.

Baranoski proposed a visual simulation method of the aurora by means of simulating the interaction between electrons and the magnetic field using particles with electrical charge\(^7\)\(^8\). In these researches, the change of the magnetic field by the influence of electrically charged particles and magnetic field generated by astronomical objects such as the earth is not considered. Therefore, it is not possible to apply these methods to the motion of the fluid on sun surface where the change in the magnetic field causes important effect.

We add magnetic field and temperature to the elements of fluid simulation based on the semi-Lagrangian advection scheme. Furthermore, we assume the fluid to be incompressible for stable simulation.

3. Solar Prominence

A prominence is a phenomenon in which a mass of plasma is ejected as far as 350,000 km from the sun’s surface.

A prominence is caused by changes in the magnetic field structure of the sun. The magnetic field rises from the surface due to the differential rotation of the sun\(^9\). A mass of plasma known as a plasmoid rises from the surface of the sun (see Fig. 2).

Thus, the time evolution of the magnetic field is an important factor in simulating the prominences.

4. Our Simulation Method

4.1 Governing equations

The plasma fluid in the sun has a very small electrical resistance and we can use an approximation known as the ideal MHD approximation. In our simulation, the following MHD equations and the time evolution of the temperature are used.

\[ \nabla \cdot \mathbf{u} = 0, \quad (1) \]

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{j} \times \mathbf{B} + 2 \Omega \times \mathbf{u}, \quad (2) \]
The equations described above consist of the Navier-Stokes equations (Equations (1) and (2)) and the Maxwell equations (Equations (3)). To solve the Navier-Stokes equations, we used the solver proposed by Stam. We propose a method to solve Maxwell equations.

To simulate the evolution of solar prominences, the velocity field, the magnetic field, and the temperature field are necessary. These physical quantities other than the magnetic field are stored in the voxels, and updated by the results of numerical analysis based on the ideal MHD equations. The magnetic field is updated by using the magnetic field lines (please see Section 4.3 for the details). The simulation space is restricted to the domain containing the prominence ejection, and represented using orthogonal coordinates. Moreover, a set of particles are generated in order to compute the evolution of the plasma fluid. The particles have positions and velocities, and are subject to Lorentz force from the magnetic field.

Fig. 3 shows the basic concept of a process at each timestep in the simulation. We define the magnetic field lines according to the initial state (please see Section 4.6 for the details) of magnetic field. Next, the velocity field is updated by calculating the Navier-Stokes equations (see Fig. 3 (b)). The magnetic field lines are advected along the updated velocity field (see Fig. 3 (c)). The particles and temperature are also advected. For each particle, the Lorentz force is calculated and particle position is updated (see Fig. 3 (d)). During the rendering process, we calculate particle colors based on the intensity of the specific spectrum.

4.3 Magnetic field lines

To update the magnetic field, we do not use the voxel representation; instead, we use line segments along the magnetic field vector. The line segments enable us to represent the magnetic field topology more efficiently than using a large number of voxels. Our method can handle the complex magnetic field, which is not possible to be represented by using coarse voxels.
In the field of physics, the magnetic field lines are only used to visualize the magnetic field and they have no meanings in the simulation. However in our method, they are used to represent the magnetic field. We define the magnetic field lines by parametric curves \( x_P \) generated from the magnetic vector field, which is given by the following equations,

\[
\frac{dx_P(\theta)}{d\theta} = B_P(x_P(\theta)), \quad (6)
\]

\[
x_P(\theta_0) = x_{P_0}, \quad (7)
\]

where \( B_P \) is the magnetic field vector of the sampling point \( P \) on parametric curves \( x_P \), which is parameterized by \( \theta \). \( B_P \) is stored in each sampling point. \( x_{P_0} \) is the starting position of the magnetic field line. These equations correspond to the streamline when the magnetic field is assumed to be a velocity field. The starting points in the stream lines are the centers of voxels at the boundary of the simulation space.

A sufficient number of magnetic field lines are calculated to reconstruct the initial magnetic field given as voxel data. The magnetic field lines are subdivided into small segment \( dx \) and the sampling points are generated. Thus, the magnetic field lines are represented by a set of connected sampling points. We use the following equation to reconstruct the magnetic field from the sampling points:

\[
B^*(x) = \sum_{P} W(x - x_P)B_P, \quad (8)
\]

where \( B^* \) is the reconstructed magnetic field, \( W \) is a kernel function defined as,

\[
W(x - x_P) = \begin{cases} 
    w \cdot \exp\left(-\frac{||x - x_P||^2}{2r^2}\right) & (||x - x_P|| \leq r) \\
    0 & (\text{otherwise}),
\end{cases} \quad (9)
\]

where \( w \) is a user-specified weighting factor, \( r \) is an influence radius. We use \( w = 0.8 \) and set \( r \) to one-tenth to the longest width of the simulation space.

Note that the definition of our magnetic field lines stated above is different from that in the physics. Our magnetic field lines are developed for efficient representation and fast reconstruction of the magnetic field, whereas the magnetic field lines appearing in physics are used for easily understandable visualization by putting more lines in regions with stronger magnetism.

### 4.4 Time evolution of magnetic field

Equation (3) for the evolution of the magnetic field is rewritten in the following way:

\[
\frac{\partial B}{\partial t} = (B \cdot \nabla)u - (u \cdot \nabla)B + u(\nabla \cdot B) - B(\nabla \cdot u). \quad (10)
\]

The third term of Equation (10) is zero according to Gauss’s law for magnetism of the Maxwell’s equations, and the fourth term is zero using Equation (1). Therefore, we can further rewrite Equation (10) as

\[
\frac{\partial B}{\partial t} + (u \cdot \nabla)B = (B \cdot \nabla)u. \quad (11)
\]

The second term of the left hand side in Equation (11) is an advection term, so the sampling points on the magnetic field lines are moved according to the velocity field. The right hand side of Equation (11) is a stretching term due to the velocity field.

### 4.5 Motion of plasma particles

We use the particles to calculate the detailed motion of the plasma including rotation motion caused by Lorentz force. The particles are also used to render the simulation results. Each particle represents a fraction of plasmoid. These particles are advected along the velocity field and their colors are calculated from the temperature field. The particles have electric charge, and are subject to the Lorentz force calculated from the magnetic field. As just advecting particles along the velocity field updated by using the coarse grid, we cannot express twist of prominences.

Therefore, we apply the Lorentz force to particles to reproduce the fine movement (see section 6 for more details). The Lorentz force affects the update of the velocity field and the motion of particles. Therefore, the equation of motion of the particles can be written as follow:

\[
\frac{dv}{dt} = \frac{q}{m}(v \times B), \quad (12)
\]

where \( v \) is the velocity vector of the particle, \( q/m \) is charge/mass ratio. \( v \) is determined by updating using Equation (12) and interpolating the voxel data \( u \).

### 4.6 Initial condition of prominence simulation

We refer to research results in astronomy\(^{10}\) to determine the initial condition of the magnetic field lines, because it is difficult to obtain real data for the magnetic field of the sun. We determine the initial magnetic field and velocity field in the normalized simulation space by using the following equation,

\[
u = (0, u \cos (\pi (x - x_{mid})), 0), \quad 0 \leq x \leq 1, \quad (13)\]
Table 1 Parameter setting of prominence simulation.

<table>
<thead>
<tr>
<th>parameter</th>
<th>meaning</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
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<td>dt</td>
<td>time step</td>
<td>0.01</td>
<td>hour</td>
</tr>
<tr>
<td>d</td>
<td>voxel width</td>
<td>3.0$\times$10$^3$</td>
<td>km</td>
</tr>
<tr>
<td>u</td>
<td>magnitude of velocity in Equation (13)</td>
<td>1.8$\times$10$^3$</td>
<td>km/h</td>
</tr>
<tr>
<td>$B_{\text{photosphere}}$</td>
<td>magnetic field strength in the photosphere in Equation (14)</td>
<td>2000</td>
<td>G</td>
</tr>
<tr>
<td>$B_{\text{corona}}$</td>
<td>magnetic field strength above the photosphere</td>
<td>26</td>
<td>G</td>
</tr>
<tr>
<td>$T_{\text{photosphere}}$</td>
<td>temperature for an environmental value in the photosphere</td>
<td>6.0$\times$10$^4$</td>
<td>K</td>
</tr>
<tr>
<td>$T_{\text{corona}}$</td>
<td>temperature for an environmental value above the photosphere</td>
<td>1.0$\times$10$^6$</td>
<td>K</td>
</tr>
</tbody>
</table>

The solar prominences are observed by extracting Hα spectrum. In the field of astronomy, the observed images (original images are in grayscale) are colored according to the intensity of plasma emission by using a programming language "IDL". Fig. 5 shows our rendering process. During the rendering process, the intensity plasma emits is calculated and converted into RGB by referring to the color table which astronomers use. We use the RGB conversion function represented by tone curve as shown in the Fig.5.

The solar prominence colors in the final images are determined from the intensity of the Hα spectrum. The visible spectrum of light from hydrogen displays four wavelengths, 410.17 nm (violet), 434.05 nm (blue), 486.13 nm (blue-green), and 656.28 nm (red). The wavelength of 656.28 nm is the Hα spectrum.

While the black body radiation can be used to calculate the intensity of solar photosphere, the Hα spectrum is mainly due to the emission process of hydrogen plasma, which is determined through quantum mechanics. The solar prominence cannot be considered as a perfectly black body, because the solar prominence is mainly composed of hydrogen plasma.

Hα spectrum is emitted due to the photoelectric effect when the energy level of hydrogen atoms moves...
from an upper level 3 to a lower level 2. This transition occurs stochastically, and its probability is known as the Einstein’s A coefficient for emission $^{12)}$. The intensity of Hα spectrum $I_{H\alpha}$ is calculated by the following equation, considering absorption and the induced emission.

$$I_{H\alpha} = \frac{h\nu_{H\alpha}L}{4\pi} \frac{N_3A_{32}}{N_3B_{32} - N_2B_{23}},$$  \hspace{1cm} (15)

where $h$ is Planck’s constant, $\nu_{H\alpha}$ is the frequency of emitted light, $L$ is the thickness of plasma, $N_i$ is the number of atoms of the energy level $i$, $A_{32}$ is Einstein’s A coefficient when the energy level moves from 3 down to 2. $B_{23}$ and $B_{32}$ are the Einstein’s B coefficients for absorption and induced emission respectively (see appendix for details). The existence probability of the atoms of energy level $i$ is calculated from Boltzmann principle$^{13)}$,

$$\frac{N_i}{N} = \frac{g_i e^{-E_i/kT}}{u(T)},$$  \hspace{1cm} (16)

where $N$ is the total number of atoms, $N_i$ is the number of the atoms of energy level $i$, $g_i$ is the degeneracy of energy level $i$ (In case of a hydrogen atom made from spinless particles, $g_i$ is $i^2$ $^{14)}$), $E_i$ is the energy level $i$, $k$ is Boltzmann’s constant. $T$ is the temperature obtained from Equation (4). $u(T)$ is called partition function and defined as the following equation $^{15)}$,

$$u(T) = \sum_i g_i e^{-E_i/kT}.$$  \hspace{1cm} (17)

6. Results

In this section, we first present a validation of our approach using the magnetic field lines, then show the simulation results of the solar prominence.

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Table 2 Numerical error between the magnetic field simulated by using magnetic field lines and that by using voxels

<table>
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<th>0.1</th>
<th>0.01</th>
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<td>0.5274</td>
<td>0.2538</td>
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<tr>
<td>0.25</td>
<td>0.2528</td>
<td>0.1296</td>
<td>0.1372</td>
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(a) unidirectional flow

<table>
<thead>
<tr>
<th>dx</th>
<th>dt</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>23.3682</td>
<td>7.4897</td>
<td>1.3763</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>11.4603</td>
<td>3.2574</td>
<td>0.5988</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>6.5817</td>
<td>2.8531</td>
<td>0.4054</td>
<td></td>
</tr>
</tbody>
</table>

(b) vortex flow
this experiment, the velocity field is fixed. We compute RMS error between the voxel data converted from magnetic field lines and the simulating data by using voxels. Fig. 6(a) shows the initial state of the experiment. In Fig. 6, the gray lines are magnetic field lines, which are initially set perpendicular to the $xz$ plane. Figs. 6(b) to (d) show the results of magnetic field lines transformed by the velocity field based on Equation (18). Tables 2(a) and (b) show the results of the experiments which are simulated under the settings of the initial velocity fields being unidirectional flow and vortex flow, respectively. The numerical value is the RMS error after 100 time steps. $dx$ is the sampling interval of magnetic field lines, shown in the ratio to the width of a unit voxel, and $dt$ is the time step of the simulation. The number of grid points is $64 \times 64 \times 64$. From Table 2, we can see that the error becomes smaller as $dx$ or $dt$ are set smaller. This error is small enough and the variance is also small. Therefore we can accurately simulate the time evolution of the magnetic field by using magnetic field lines instead of the voxels. Moreover, our method can handle the complex magnetic field including the twist of magnetic field lines, which is not possible to be represented by using coarse voxels. Fig. 6(d) shows magnetic field lines after 90 time steps of the simulation with the velocity field initialized to form a vortex. The twist of magnetic field lines can be simulated as shown in Fig. 6(d).

Fig. 7 shows the relationship between numerical errors and the number of magnetic field lines while $dx$ and $dt$ are fixed.

6.2 Simulation results of the solar prominence

We map the texture based on an observed image on the sun’s surface and the corona is also displayed by mapping the texture based on a Gaussian distribution.

We compare the magnetic field calculated by using magnetic field lines with results using volume data. Fig. 8 shows the comparison between results using our method and voxel based simulation to calculate the time evolution of magnetic field. Our method can simulate the complex behavior of the plasma including rising and twisting of the plasma around magnetic field lines. This twisting effect is the result of adding the Lorentz force to the particles.

We compare the result of the emission process and the result of the black body radiation. The intensity

Fig. 8 Comparison between results using voxel based simulation (top) and using our method (middle). Bottom shows the results with Lorentz and Coriolis forces applied to our method.

Fig. 9 Comparison of the intensity calculations. Calculation result of the intensity based on black body radiation (top) and emission process of hydrogen plasma (bottom).
$B_\lambda$ of the black body radiation is calculated from the Planck distribution function.

$$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1},$$

(19)

where $\lambda$ is the wavelength and $c$ is the speed of light. Fig. 9 shows the results of the intensity by the black body radiation and the emission process. It is necessary to consider the emission process when the temperature of the objects is high, such as plasma. Fig. 10 shows the prominence simulated by our method, where the temperature of the objects is high, such as plasma.

The resolution of the grid is $64 \times 128 \times 64$. We use 10 magnetic field lines and each magnetic field line is subdivided into 960 sampling points. The average computation time per a single timestep is 3 seconds on a standard PC (CPU: Core i7 3.2GHz, RAM: 12.0GB). We can synthesize the shape of the prominence that is similar to the real prominence by adding Lorentz and Coriolis forces as the external force.

Fig. 11 shows appearance of the prominence when viewed from two different viewpoints. Fig. 12 shows the comparison between the result using our method and a satellite image taken by NASA.

Fig. 13 shows images of the simulation results of the entire sun. Figs. 10 (a) to (d) are the results for 100 to 190 minutes. The evolution of the prominence depends on the latitude, since the Coriolis force is determined by the latitude. In these simulations, one timestep corresponds to approximately 2 minutes.

7. Conclusions and Future Work

We have proposed an efficient method for the time evolution of a magnetic field, and have shown its application to solar prominences. Taking into account the MHD equations, we can simulate the behavior of a plasma fluid. Our method is able to render the sun by calculating the specific wavelength actually observed.

For future work, we are planning to implement our method on the GPU and to develop a stable simulation method including a magnetic reconnection.

As we focused on physical-based simulation of prominences, we merely used simple texture mapping to represent the corona and supergranulation (a particular pattern on the sun’s surface). We plan to propose models for these phenomena as well as for flares (the explosive phenomenon observed above the photosphere).

References


Appendix A  Einstein’s A coefficient

The Einstein’s A coefficient is the spontaneous transition probability from an energy level to another level and is calculated by means of perturbation the-
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Fig. 10 Simulation results of erupting prominence. (a) to (d) are final images created by our method. (e) to (h) are distribution of magnetic field lines corresponding to (a) to (d), respectively.

Fig. 11 Result images of changing the viewpoint.

Fig. 12 Comparison between our result and the actual phenomena above the sun.

ory of quantum mechanics. We use the perturbation term $H'(x, t)$ defined by Equation (A1). $H(t)$ is a time-dependent Hamiltonian. The Einstein’s A coefficient is given by Equation (A2) which can be induced from Schrodinger equation.

$$H(t) = H_0 + H'(x, t) = H_0 + x^2 e^{-t}, \quad (A1)$$

$$A_{32} = \frac{4\pi^2}{h} \left| \int_{-\infty}^{\infty} \langle n = 2 | H'(x, t) | n = 3 \rangle e^{i\omega_{32}t} dt \right|^2, \quad (A2)$$

where $H_0$ is the unperturbed Hamiltonian, $\omega_{32} = 2\pi(E_3 - E_2)/h$, $| n = i \rangle$ is a vector of the wave function $f_i(x)$ that corresponds in energy level $i$. $E_i$ is the energy level $i$ and $h$ is Planck’s constant.
We can obtain $B_{ij}$ and $B_{ji}$ in Equation (15) from $A_{ij}$ as follows,

$$B_{ij} = \frac{c^3}{8\pi h \nu_{ij}^3} A_{ij},$$  \hspace{1cm} (A3)

$$B_{ji} = \frac{g_i}{g_j} B_{ij}.$$  \hspace{1cm} (A4)

In Equation (A3), $\nu_{ij}$ is the frequency of emitted light when the energy level moves from $i$ down to $j$. The above Equations (A3) and (A4) hold at any temperature\textsuperscript{16}. 

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