Lighting Design in Interreflective Environments Using Hopfield Neural Networks

Kentaro TAKAHASHI, Kazufumi KANEDA, Takeshi YAMANAKA, and Hideo YAMASHITA
Faculty of Engineering
Hiroshima University
1-4-1 Kagamiyama, Higashi-hiroshima, 724 JAPAN

Eihachiro NAKAMAE
Department of Management and Information Sciences
Hiroshima Prefectural University
562 Nanatsu, Shoubar, 727, JAPAN

Tomoyuki NISHITA
Faculty of Engineering
Fukuyama University
985 Sanzo, Higashimura-cho, Fukuyama, 729-02 JAPAN

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ABSTRACT

This paper proposes a method for calculating luminous intensity distributions of multiple light sources taking into account both direct and interreflected light when a desired luminance distribution in a space is given. This inverse lighting problem is useful for designing rooms and tunnels. In our approach, a luminance distribution is specified instead of an illuminance distribution, because it is closely connected to the appearance of rooms and is used for lighting design in a tunnel. To calculate the intensity distributions of light sources quickly and robustly, the property of Hopfield neural networks that their energy converges to a minimum is exploited. The proposed method should greatly facilitate lighting design when used with rendering techniques such as the radiosity method. Several examples including lighting design in a tunnel are shown to demonstrate the usefulness of the proposed method.

Keywords and Phrases: Lighting Design, Luminous Intensity Distribution, Luminance Distribution, Neural Network, Inverse Problem, Interreflection, Radiosity

1. Introduction

There are two approaches to lighting design. One is a method of calculating a luminance distribution inside a room when given light sources with fixed luminous intensity distributions. This approach has been widely used in lighting design and computer graphics is one of the most effective methods to visualize the lighting effects.

To render the lighting effects in a room, a radiosity method, a lighting model taking into account interreflection of light between surfaces, was proposed by Cohen et. al.\textsuperscript{1} and Nishita et. al.\textsuperscript{2}. Many methods have been developed for increased realism of the generated images and decreased calculation time. In particular, general radiosity methods considering specular reflection\textsuperscript{3}, a global illumination method\textsuperscript{4}, a zonal method for rendering light interactions between surfaces and volumes\textsuperscript{5}, and a hybrid method of radiosity and ray tracing\textsuperscript{6} were proposed for improving the image quality. Methods for accelerating calculation speed\textsuperscript{7,8,9} and for calculating
more accurate luminance distributions\textsuperscript{10,11,12} have also been proposed.

These methods are able to calculate accurate luminance distributions. However, in order to obtain a desired luminance distribution, they require a trial and error process of changing the locations and luminous intensity distributions of the light sources. Therefore, this first approach very often requires the help of experts in lighting design. The other approach, proposed in this paper, is calculating the number, position, and luminous intensity distribution of light sources when the luminance distribution is given (we call this the inverse problem in the following discussion). This approach is quite important for many cases of lighting design: (1) Tunnels: For driving safely and smoothly, it is important that there be a constant luminance distribution on the road surface and a well-balanced luminance distribution on the walls, the ceiling, and cars ahead\textsuperscript{15}. (2) Indoor baseball fields: Both the ground and ball should have appropriate luminance. (3) Show windows: Attractive lighting effects including color of light are required. (4) Work spaces: An appropriate, constant luminance distribution is required for working safely.

A method of solving the inverse problem was developed\textsuperscript{14}, but only takes into account direct light arriving from light sources. It is thus useful only for designing street lamps, and does not apply well to indoor lighting design because of lack of interreflection of light considerations.

Our goal is to develop a method for calculating the number and luminous intensity distribution of light sources that will generate a desired luminance distribution including spectrum of light. The method makes it possible for a designer with limited experience in lighting apparatus to design lighting effects easily. However, the second approach sometimes gives a light source with unrealizable luminous intensity distribution, and the obtained luminous intensity distributions may be restricted by cost of lighting apparatus if each light source has different luminous intensity distribution. In practice, trial and error is still required, even if we use both the traditional and the proposed approaches for lighting design. However, using the second approach, lighting design becomes much easier than using only the first approach.

This paper proposes a method of lighting design for interreflective environments that belongs to the category of the second approach. When the geometry of a room, reflectivity of the surfaces in the room, and luminance distribution including both brightness and hue are given, there are four problems on light sources: number, position, luminous intensity distribution, and spectrum; it is thus required to solve a multi-variable problem.

In this paper, we solve the inverse lighting design problem under the following conditions as initial steps to address the problem.

(1) The analysis space is two dimensional.

(2) The number and position of light sources are given. That is, when a desired luminance distribution is given, luminous intensity distributions of each light source are calculated.

(3) All surfaces are perfectly diffuse. Each surface may have different diffuse reflectivity, and interreflection of light between surfaces is taken into account.

(4) The light sources are white with a uniform spectral distribution.

The property of Hopfield neural networks that their energy converges to the minimum value\textsuperscript{19}, is taken advantage of to achieve fast, robust calculation of the luminous intensity distribution of light sources that yield the desired luminance distribution. One of the advantages of the neural network solution is the potential ability to calculate the luminous intensity distribution in real time; if each neuron of the network is assigned to one processor of a parallel computer that has a large number of processors, in the near future designers can hope to design lighting effects interactively simply by changing the desired luminance distribution.

In this paper, a method for solving the inverse lighting design problem using neural networks is proposed after explaining the energy minimum property of Hopfield neural networks. To investigate the accuracy of the proposed solution of the inverse problem, the proposed method is applied to lighting design in two dimensional space. Lighting design for a tunnel also demonstrates the usefulness of the proposed method.

2. Minimizing Energy of Hopfield Neural Networks

A Hopfield neural network is a network that has mutual branches between all neurons as shown in Fig. 1, where the branch from neuron j to neuron i has weight $T_{ij}$. Neuron i has an internal value, $U_{i}$, that is governed by the output values of the other neurons, $V_{j}$, and the bias of neuron i, $l_{i}$ (see Fig. 2):

![Fig. 1 Hopfield neural network](image-url)
\[
\frac{dU}{dt} = \sum_{j=1}^{N} T_{ij} V_j + I_i \tag{1}
\]

Where \(N\) is the number of neurons in the network, and \(t\) expresses time. The output value of neuron \(i\), \(V_i\), is given by using a neuron input/output function, \(G\):

\[
V_i = G(U_i) \tag{2}
\]

In most cases, the following sigmoid function is used as the function \(G\):

\[
G(U_i) = \frac{1}{2}(1 + \tanh(U_i)) \tag{3}
\]

When the weights between the neurons are symmetric, i.e., \(T_{ii} = T_{pi}\), the energy of the network is defined by the following equation\textsuperscript{15}:

\[
E = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} T_{ij} V_i V_j - \sum_{i=1}^{N} I_i V_i \tag{4}
\]

The equation is referred to as "energy", although it does not represent the real energy of any physical system. When the time passed, the energy of a Hopfield neural network converges to the minimum. This is because the first derivative of the energy function versus time is always zero or negative as \(G(U)\) (Eq. 3) is a monotonically increasing function\textsuperscript{15}:

\[
\frac{dE}{dt} = -\sum_{i=1}^{N} G'(U_i) \left( \frac{dU_i}{dt} \right)^2 \tag{5}
\]

Appropriate initial values are established, the network starts, then the energy of the network becomes smaller, and finally reaches its minimum.

In this paper, the minimum energy property of Hopfield neural networks is applied to solving the problem of calculating luminous intensity distribution of light sources that give a desired luminance distribution. Namely, a Hopfield neural network is constructed whose energy is the sum of the squares of the differences between the desired luminance and the actual luminance of surfaces lit by the light source. Running the network until its energy reaches the minimum calculates the luminous intensity distributions of light sources that give a desired luminance distribution.

3. Constructing the Neural Network

As described in the previous section, a Hopfield neural network is constructed to solve the inverse lighting design problem, and the energy of the network is the sum of the squares of the differences between the desired luminance and the luminance of surfaces lit by light sources. In this section, we propose a method for constructing a network for calculating luminous intensity distributions of light sources using a two-dimensional lighting model.

Assume that a point light source with luminous intensity distribution \(V(\alpha)\) is present in two-dimensional space enclosed in a polygon as shown in Fig. 3. The edges of the polygon, corresponding to surfaces in three-dimensional space, are divided into small segments, corresponding to patches, and the radiosity of all segments is expressed by \(M = [M_1, M_2, \ldots, M_m]^T\) (where \(m\) is the number of segments), and the radiosity of segment \(i\), \(M_i\), is given by the following luminous flux transfer equation:

\[
M_i = M_{0i} + \rho_i \sum_{j=1}^{m} M_j F_{ij} (i = 1, \ldots, m), \tag{6}
\]

Where \(M_0\) is an emission value for segment \(i\), that is, it denotes the average luminance of segment \(i\) due to direct light, \(\rho\) is the reflectivity of segment \(i\), and \(F_{ij}\) is a form factor between segment \(i\) and \(j\) in two-dimensional space\textsuperscript{16}. Eq. 6 can be rewritten in matrix form:

\[
KM = M_0 \tag{7}
\]

![Fig. 3: Two dimensional lighting model for calculating a luminous intensity distribution of a light source.](image)
Where \( K \) is a \( m \times m \) matrix whose elements are \( K_{ij} = \delta_{ij} - \rho_F \) (\( \delta_{ij} = 1 \) when \( i = j \), and 0 otherwise). Matrix \( K \) is a constant matrix depending on the geometry of the polygon consisting of segments and the reflectivity of each segment.

In two dimensional space, illuminance is in inverse proportion to distance from the light source, because light travels only in a surface. Therefore, the average illuminance of segment \( i \), \( M_i \), due to direct light is given by the following equation (see Fig. 3):

\[
M_{0i} = \frac{\rho_i}{L_i} \int_{L_i} \frac{\cos \theta(x)}{r(x)} V(\alpha(x)) \, dx
\]

(8)

Where \( x \) is an arbitrary point on segment \( i \), \( \theta(x) \) is the angle between the perpendicular of segment \( i \) and the vector from the light source to point \( P \), \( r(x) \) is the distance between the light source and point \( P \), \( \alpha(x) \) is the vertical angle of the light vector toward point \( P \), and \( L_i \) is the length of segment \( i \).

The luminous intensity distribution, \( V(\alpha) \), is discretized into \( n \) luminous intensity vectors, \([V = V_1, V_2, ..., V_n]^T\), for the angle \( \alpha \) as shown in Fig. 4. If luminances at \( m \) sampled points on segment \( i \) are calculated, the average luminance of the segment is approximated by using trapezoidal integration:

\[
M_{0i} = \frac{\rho_i}{m'} \sum_{k=1}^{m'} T_k \frac{\cos \theta_k}{r_k} V_k
\]

(9)

Where \( T_k \) is 1/2 when \( k = 1 \) or \( m' \), and 1 otherwise. The luminous intensity vector \( V_k \) toward sampled point \( P \), is estimated by interpolating two discretized luminous intensity vectors \( V_i \) and \( V_{i+1} \) as shown in Fig. 4:

\[
V_k = \frac{\alpha'_{j+1} V_j + \alpha'_{j} V_{j+1}}{\alpha'_j + \alpha'_{j+1}}
\]

(10)

Substituting Eq. 10 for Eq. 9, the following equation in matrix form is obtained:

\[
M_i = H V,
\]

(11)

Where \( H \) is a \( m \times n \) constant matrix whose elements depend on the geometry of the light source and the segments.

Using Eqs. 7 and 11,

\[
M = K^{-1} H V,
\]

(12)

\[
M = A V,
\]

(13)

Where \( A = K^{-1} H \), because both matrices \( K^{-1} \) and \( H \) are constant matrices whose elements depend on the geometry of the segments and the light source.

Now, let us focus on the objective function, \( O \), of the inverse problem. If the desired luminance distribution is \( Q = [Q_1, Q_2, ..., Q_n]^T \), and luminance distribution due to the light source with a luminous intensity distribution, \( V_i \), is \( M_i \), then the objective function, \( O \), is expressed by the following equation:

\[
O = \sum_{i=1}^{m} (Q_i - M_i)^2
\]

(14)

The luminous intensity distribution of a light source that gives the desired luminance distribution is obtained by minimizing the objective function expressed in Eq. 14. Using Eq. 13, Eq. 14 is modified as follows:

\[
O = \sum_{i=1}^{m} \left( Q_i^2 + \sum_{k=1}^{n} \sum_{l=1}^{n} a_{ik} a_{il} V_j V_k - 2 \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ik} Q_j V_k \right)
\]

\[
= \sum_{i=1}^{n} Q_i^2 + \sum_{j=1}^{n} \left( \sum_{k=1}^{n} a_{ik} V_j \right)^2 - 2 \sum_{j=1}^{n} \left( \sum_{k=1}^{n} a_{ik} Q_j \right) V_j
\]

(15)

Here,

\[
T_{jk} = -2 \sum_{i=1}^{n} a_{ij} a_{ik}
\]

and

\[
I_j = 2 \sum_{i=1}^{n} Q_i a_{ij}
\]

(16)

(17)

then, the objective function is expressed by the following equation:
\[ O = \sum_{i=1}^{m} Q_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij} V_j V_j - \sum_{i=1}^{m} I_i V_i \]
\[ = \sum_{i=1}^{m} Q_i^2 + E \]  \hspace{1cm} (18)

The first term of Eq. 18 is constant, and the sum of the second and third s equal to the energy function of the Hopfield neural network expressed in Eq. 4. 

From Eq. 18, the network to solve the inverse lighting design problem consists of the discretized luminous intensity vectors, \( V_i \), as its neurons. The weight of the branch connecting neurons \( k \) and \( j \) is given by Eq. 16, and the bias of neuron \( j \) is given by Eq. 17. When the network reaches a steady state, the output value, \( V_i \), of the neurons yields the luminous intensity distribution of the light source giving the desired luminance distribution, \( Q \).

4. Calculating the Luminous Intensity Distribution

4.1 Single light source

To investigate the accuracy of the proposed method, a luminance distribution is calculated by using the first approach described in Section 1, and then the distribution is run through the proposed method. The luminous intensity distribution obtained from the proposed method is compared to the original one.

Let’s consider a model of a room enclosed within four walls whose reflectivity is 0.6 (see Fig. 5 (a)). First, setting a light source with the luminous intensity distribution shown in Fig. 5 (a), the luminance distribution on the walls is calculated. False color is used to indicate luminance: red is bright and blue is dark. Then, giving the luminance distribution to the proposed method as a desired luminance distribution, a luminous intensity distribution of the light source is calculated. Fig. 5 (b) shows the result. The resulting luminous intensity distribution is almost equal to that of Fig. 5 (a). The luminance distribution shown in Fig. 5 (b) is calculated by using the light source with the resulting luminous intensity distribution. Fig. 5 (c) and (d) show the relative and absolute error distributions of luminance in Fig. 5 (b), respectively, compared to that of Fig. 5 (a). The average and maximum errors are 0.29% and 1.88%, respectively. The number of segments is 52 and the calculation time to obtain luminous intensity distribution using a neural network is 9 seconds on an IRIS Indigo R4000.

Fig. 6 shows the case of a light source with large luminous intensity vectors in a specific direction such as a spot light. Reflectivities of the wall, floor, and ceiling are set to 0.7, 0.5, and 0.4, respectively. Each sub-figure of Fig. 6 corresponds to that of Fig. 5. Even in the case of the spot light and different reflectivities, the luminous intensity distribution is calculated accurately.

In the case of Figs. 5 and 6, the initial luminous intensity distribution of the light source, i.e. the initial internal values of neurons, is set to different distributions such as circles and ellipses with different radii. For all of the initial distributions, the neural network converges to the same steady state. That is, the results are the same as those of Figs. 5 (b) and 6 (b), respectively.

4.2 Multiple light sources

To extend the proposed method described in the previous section to multiple light sources, the luminance of each segment due to direct light is calculated for all of the light sources using Eq. 10. The number of elements in the luminous intensity distribution, \( V \), becomes the sum of discretized luminous intensity vectors of each light source, \( N \), and the constant matrix, \( H \), in Eq. 11 becomes a \( m \times N \) matrix. As the result of increasing the number of light sources, the number of neurons in the network to solve the inverse problem becomes \( N \).

Fig. 7 (a) shows a desired luminance distribution to calculate luminous intensity distributions of multiple light sources. The reflectivity of the all walls is 0.6. The luminance of the floor and ceiling is uniform, and the luminance of the walls changes linearly from floor to ceiling. The resulting luminous intensity distribution are shown in Figs. 7 (b) to (e). For these figures, different initial distributions of luminous intensity are given; the initial distribution curves are shown in gray. Table 1 shows relative and absolute errors of luminance due to the light sources with the resulting luminous intensity distribution. For all of these cases, luminance distributions due to the obtained light sources are very similar to the desired distribution. That is, in the case of multiple light sources, there exist different luminous intensity distributions satisfying the desired luminance distribution.

### Table 1: Relative and absolute errors of luminance compared with Fig.7(a)

<table>
<thead>
<tr>
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<th>Fig.7(b)</th>
<th>Fig.7(c)</th>
<th>Fig.7(d)</th>
<th>Fig.7(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative error (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>2.40</td>
<td>2.55</td>
<td>2.59</td>
<td>2.60</td>
</tr>
</tbody>
</table>

| Absolute error (cd/m) |          |          |          |          |
| average | 4.55     | 4.65     | 4.55     | 4.75     |
| maximum | 14.1     | 14.1     | 14.1     | 14.2     |

Comparing initial luminous intensity distributions with the obtained ones, the following is clear. If the luminance on a segment due to multiple light sources is smaller than desired, the luminous intensity vectors of the clos-

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Note 1 Fig. 5 – Fig. 10 are on pp. 5–8.
est light source to the segment tend to become larger. In the opposite case, that is, luminance is larger than desired, the luminous intensity vectors of the closest light source to the segment tend to become smaller. That is, the neural network tends to adjust the luminous intensity distribution of the light source that has the largest influence on the luminance distribution.

Both the luminous flux of a light source and the complexity of the luminous intensity distribution are important criteria in lighting design. Therefore, the best luminous intensity distribution should be selected from several candidates obtained by changing the initial value, and the selection should be based on comprehensive criteria such as the shape of the luminous intensity distribution curve and the energy emitted from the light source. In the specific case of finding a luminous intensity distribution most similar to a specified distribution, it is best to use the specified distribution as the initial value.

5. Lighting Design for a Tunnel

If a very long, straight tunnel is equipped with linear light sources, the lighting effects can be designed using a two dimensional model of a perpendicular cross section of the tunnel. In this case, the obtained luminous intensity distribution is the sum of light arriving from every point on the linear light source. Therefore, to get a real luminous intensity distribution of the linear light source, the obtained luminous intensity should be compensated.

The resulting luminous intensity distributions are shown in Figs. 8-10 (b). The luminance distributions in these figures are calculated by using the resulting luminous intensity distributions. Figs. 8-10 (c) show the error distributions of the luminance compared with the luminance distributions of Figs. 8-10 (a). Average and maximum errors are also shown in Table 2. For each case, it took about 13 seconds to calculate the luminous intensity distributions using a neural network on an IRIS Indigo R4000.

In Figs. 8-10 (d), the resulting luminous intensity distributions are modified manually into simplified distributions for their implementation, because the obtained distributions are too complex for actual use. Error distributions of the luminance due to the light sources with the modified luminous intensity distributions are shown in Figs. 8-10 (e).

In Figs. 8-10 (f), the tunnel lit by the linear light sources with the modified luminous intensity distributions is rendered using the first approach described in Section 1. The brightness on the walls is gradually changed in Fig. 8 (f), is uniform in Fig. 9 (f), and in Fig. 10 (f), the brightness of all surfaces is almost uniform though the brightness on the walls and road is less than in Fig. 9 (f). It is difficult to create such lighting effects using only the first approach. Using the proposed approach in lighting design together with the traditional one, the desired lighting can be easily designed.

6. Conclusions

A method of solving the inverse lighting design problem using a neural network is proposed; when a desired luminance distribution in a space is given, luminous intensity distributions of multiple light sources are calculated taking into account both direct and interreflected light. The proposed method makes it possible for a designer with little experience in lighting apparatus to easily design lighting effects in an interreflective closed space.

There are still many problems to be solved in the inverse problem of lighting design. In this paper, we used a two dimensional model in order to make the basic points of the inverse problem clear. In practical use, a three dimensional model is required. It would be easy to extend the proposed method to a three dimensional model; the luminous intensity distribution curve described in Section 3 is extended to a solid, and the solid is discretized into luminous intensity vectors in polar coordinates. Improving the proposed method to handle surfaces with specular reflection will also make the method more useful.

To generalize the inverse lighting design problem, not only luminous intensity distributions but also the suitable number and positions of light sources should be calculated from a desired luminance distribution. For lighting design taking into consideration the color of light, such as for show windows, it will be necessary to de-

<table>
<thead>
<tr>
<th>Table 2: Average and maximum errors of luminance compared with the desired luminance distributions</th>
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velop a method of solving the inverse problem taking into account the spectra of light sources.

6. Acknowledgment

The authors wish to thank Muneki Shimada for generating several images in a tunnel.

References

(14) Hamabe T.: Patent (a utility model) applied for
Fig. 5 Calculating a luminous intensity distribution of a single light source. \( \rho \) is reflectivity. \( \text{Cd/m} \) is used as the unit of luminance because of two dimensional space. 
(a) desired luminance distribution 
(b) the obtained luminous intensity distribution and luminance distribution. 
(c) relative error distribution. 
(d) absolute error distribution.

Fig. 6 Calculating a luminous intensity distribution of a spot light. \( \rho \) is reflectivity. \( \text{Cd/m} \) is used as the unit of luminance because of two dimensional space. 
(a) desired luminance distribution. 
(b) the obtained luminous intensity distribution and luminance distribution. 
(c) relative error distribution. 
(d) absolute error distribution.
Fig. 7 Calculating luminous intensity distributions of multiple light sources. $p$ is reflectivity. The initial luminous intensity distribution of each light source is shown in gray. The resulting luminous intensity distributions are shown in blue, green, or orange.
(a) desired luminance distribution. (b) the resulting luminous intensity distribution and luminance distribution (the initial distribution is a point). (c), (d) and (e) the resulting luminous intensity distribution and luminance distribution.

Fig. 8 Examples of lighting design in a tunnel (case 1). The initial luminous intensity distribution of each light source is shown in gray. The resulting luminous intensity distributions are shown in blue or orange.
(a) desired luminance distribution. (b) the resulting luminous intensity distribution and luminance distribution. (c) relative error distribution of luminance shown in Fig. 8 (b). (d) the modified luminous intensity distribution and luminance distribution. (e) relative error distribution of luminance shown in Fig. 8 (d).
Fig. 9  Examples of lighting design in a tunnel (case 2). The initial luminous intensity distribution of each light source is shown in gray. The resulting luminous intensity distributions are shown in blue or orange.
(a) desired luminance distribution.  (b) the resulting luminous intensity distribution and luminance distribution.  (c) relative error distribution of luminance shown in Fig. 9 (b).  (d) the modified luminous intensity distribution and luminance distribution.  (e) relative error distribution of luminance shown in Fig. 9 (d).

Fig. 10  Examples of lighting design in a tunnel (case 3). The initial luminous intensity distribution of each light source is shown in gray. The resulting luminous intensity distributions are shown in blue or orange.
(a) desired luminance distribution.  (b) the resulting luminous intensity distribution and luminance distribution.  (c) relative error distribution of luminance shown in Fig. 10 (b).  (d) the modified luminous intensity distribution and luminance distribution.  (e) relative error distribution of luminance shown in Fig. 10 (d).
Fig. 8 (f), Fig. 9 (f), and Fig. 10 (f): Scenes in a tunnel lit by the light sources with luminous intensity distribution shown in Fig. 8 (d), Fig. 9 (d), and Fig. 10 (d), respectively.