An Algorithm for Half Toned Representation of Three Dimensional Objects

Tomoyuki Nishita* and Eihaciro nakamae**

1. Introduction

The problems concerning hidden line elimination[1)~4)] and half toned presentation [5)~10)] have to be solved to realistically display three dimensional objects on two dimensional planes. However, these problems are usually neccessary considerably complex input data and programs, and a great amount of computing time. Therefore, from the view points of man-machine systems, these problems have not been solved sufficiently. The authors presented an algorithm for hidden line elimination of polyhedra which were divided into proper convex polyhedra, and showed that the program based on this algorithm has ability of saving computing time.[4)] Expanding the algorithm mentioned above, a half toned presentation algorithm for three dimensional objects illuminated with parallel light are proposed, and some examples are expressed to establish the usefulness of this algorithm. The merits of this algorithm are as follows; 1) as an object is divided into proper convex polyhedra and they are standardized, the preparation for input data is very easy and the data errors can be reduced compared with traditional methods, 2) the procedures for changing sizes and positions of convex polyhedra are very simple and the some of them can be generated automaticaly, 3) the algorithm of half toned representation is simple, and the computing time is saved by making a good use of the property of the contour lines of convex polyhedra.

2. Algorithm

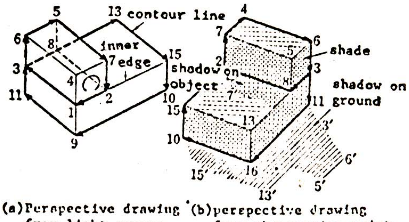
The technical terms used in this paper are the same as that of the reference [4]. The algorithm is based on the following assumption: 1) the light source is parallel, 2) the illuminance caused by reflex light from objects are eliminated, 3) reflection coefficients are constant due to the ability of the half toned representation, and 4) the contrast caused by the distance from an observation point to objects is also eliminated.

This paper first appeared in Japanese in Joho—Shori (Journal of the Information Processing Society of Japan), Vol. 15, No. 1 (1974), pp. 18~25.

* Toyo Kogyo Co., Ltd., Hiroshima, Japan
** Faculty of Engineering, Hiroshima University, Hiroshima, Japan.

Curved objects and surfaces are approximated by a set of convex polyhedra and planes.

The algorithm for half toned representation is as follows. First, a light source is assumed as the first observation point, and the shadows of objects are obtained for the light source. That is, the vertices on convex polyhedra are transposed on the coordinates of a perspective plane for the light source. Thus, the back faces for the light source become to be shade. The inner edges and contour lines of the convex polyhedra are obtained by using the directions of the planes. After that, the intersections of the contour lines are searched. The part of the intersection is the shadow on a convex polyhedron by the other one located nearer than it from the light source as shown in fig.1(a). These contour lines also make up the contour line of the shadow on the ground. Therefore, they are obtained by transposing the coordinates of the vertices on the perspective plane. Second, an observer's observation point is assumed as the second observation point and the half tone of the objects are obtained. That is, the vertices are transposed on the coordinates of a perspective plane for the observation point, and the directions of planes are obtained. After that, the inner edges and contour lines of the convex polyhedra are also obtained. Then, the intersections concerning the contour lines of the convex polyhedra on the perspective plane are calculated, and if some of them intersect each other, the relationship between the locations of the polyhedra are stored. Last, on the perspective plane the convex polyhedra and the shadows are scanned like television scanning, and the half tone on each point on scanning lines is calculated and displayed.



(a)Perspective drawing from light source. (b)perspective drawing from observation point.

Fig. 1 Perspective drawings.

2.1 Classification of Edges

As a plane observed from an outside is defined as a clockwise, and is convex, if the plane, S , is composed of the vertices, [P_1, P_2, P_3, ...] , its normal vector, N , is expressed by (P_1-P_2)×(P_3-P_2). Therefore, whether the plane is oriented to the observation point, $P_v(X_v,Y_v,Z_v)$, or not is judged by the sign of the following equation,

$$\Delta_{pv} = N \cdot (P_v-P_1). \qquad (1)$$

On the other hand, for the light source expressed by $L(X_1,Y_1,Z_1)$, the judgement is carried out by equation (2), because it is located at infinity.

$$\Delta_1 = N \cdot L \; . \tag{2}$$

However, for both of the light source and the observation point, if the direction of P_1, P_2 and P_3 forming a plane is clockwise on the perspective plane, the plane is front faced. Then, the direction of the plane is judged by means of the sign of the following equation using the coordinate on the perspective plane,

$$D = -\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \begin{cases} > 0 \text{ (front face)} \\ \leq 0 \text{ (back face) .} \end{cases} \tag{3}$$

From these relationship, the inner edges and contour line segments of each convex polyhedron are obtained. Therefore, the contour lines of polyhedra are obtained by the use of the directions of the contour line segments.

2.2 Location Relationship between Two Convex Polyhedra

Whether two convex polyhedra overlap or not when they are observed from an observation point or a light source is judged by the use of the intersection relationship with the contour lines of the polyhedra on the perspective planes . In case the contour lines intersect each other, it is neccessary to judg which one is nearest to the observation point. This is carried out by using the location relationship between the points existing on one of the intersections of the contour lines on the convex polyhedra. On the other hand, in cace a contour line is surrounded by the other, the judgement about the distance from the observation point is carried out by means of eq. (1) concerning the vertex, P, on the surrounded convex polyhedron and the front face, S, surrounding P and belonging to the surrounding convex polyhedron.

2.3 Shadow and Illuminance on Plane

In case the contour lines of two convex polyhedra intersect each other on the perspective plane concerning a light source, a shadow occures on the planes belonging to a hidden polyhedron. Assuming that the hidden polyhedron is V_j and the hidding one is V_i, the contour line of V_i makes a shadow on the planes belonging to V_j. The algorithm to obtain the domain of the shadow is explained by fig.2. First, one of the intersections of the

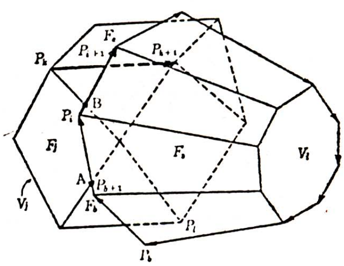


Fig.2 Search for shadow on plane.

contour lines is obtained. Then, the intersection of the contour line segment, $[P_b, P_{b+1}]$, belonging to V_i and entering to V_j, and one of the contour line segments belonging to the plane, F_b, shown in fig.2 is searched. In case of no intersection, the neighbor contour line segment of $[P_b, P_{b+1}]$ is chosen and an intersection is searched in the same way. These procedures are continued until one of the contour line segments of V_i intersects with one of the contour line segments or inner edges belonging to F_b. If one of the contour line segments of V_i intersects with an inner edge, the plane neighboring F_i and including the intersection is searched and the location of the intersection is also searched in the same way. These procedures are continued until the contour line of V_i leaves V_j. In case polyhedra are observed from an observation point, the coordinate of the shadows locating on the planes of polyhedra are obtained as the points on three dimension coordinate, and transposed on a perspective plane.

By the method mentioned above, the plane surrounded by the contour lines of V_i, e.g. F_s in fig.2, can not be obtained. In this case, if an arbitrary vertex, P_1, belonging to the plane which does not intersect with the contour line is surrounded by it, it is judged that the whole plane is in a shadow.

It is required to obtain the illuminance of the front faces concerning the observer's observation point. Assuming that a unit light vector is expressed by $L(X_L, Y_L, Z_L)$, as the illuminance of each plane is in proportion to cosine of the light and the normal vector of the plane, N, the following relationship is established,

$$\cos\theta = (L \cdot N) / (|L| \cdot |N|) . \quad (4)$$

Therefore, the tone of the plane H is expressed by

$$H = H_{max}(1 - k\cos\theta) , \quad (5)$$

where H_{max} is maximum depth and k is a refrection coefficient. Obviously, the tone expressing shadows is H_{max}.

2.4 Scanning Algorithm

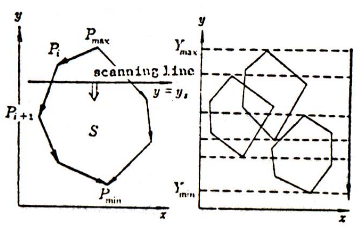


Fig.3 Scanning.

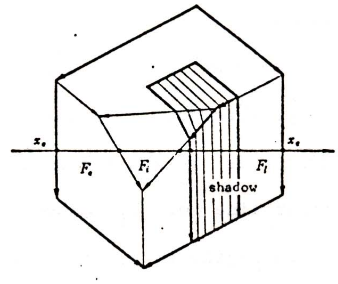


Fig.4 Tone on scanning line.

The tone at each point is calculated by scanning from top to bottom on a perspective plane. Since all of the shapes of contour lines and shadows on the ground and planes are composed of a convex polygon, the judgement about intersection is easily carried out by the following procedures. Let P_{max} be the maximum point in the vertices of a convex polygon, S, concerning y axis, P_{min} the minimum point, and y_{max} and y_{min} the values of y axis coresponding P_{max} and P_{min} as shown in fig.3, respectively. Assuming that the value of a scanning line is y_s concerning y axis and $y_{min} \leq y_s \leq y_{max}$, the scanning line and S intersect each other and two intersections exist on clockwise and counter clockwise line segments which are oriented from top to bottom. Therefore, each of them is obtained independently.

The planes locating on the scanning line are searched by means of the following procedures. Assuming that the points, x_e and x_l, are the intersections of a convex polyhedron and the scanning line, and it enters at x_e and leaves x_l as shown in fig.4, in case of $F_e \neq F_l$, the intersection of the inner edge belonging to F_e and the scanning line is easily obtained. Using this inner edge, the neighbor plane F_i is searched, and the other intersection of it and the scanning line is obtained. The same procedures mentioned above are continued until the scanning line leaves the plane F_l.

When many polyhedra and shadows overlap on a scanning line, it is neccessary to determine which of these should be visualized. First, all of the shadows on the ground are memorized. After that, the tones and shadows on the planes are memorized in order of long distance from an observation point. One of examples is shown in fig. 5 (a). However, in the case of fig.5 (b), to judge the order is somewhat difficult. This trouble is solved by means of that the ordering is carried out about only the polyhedra on the scanning line.

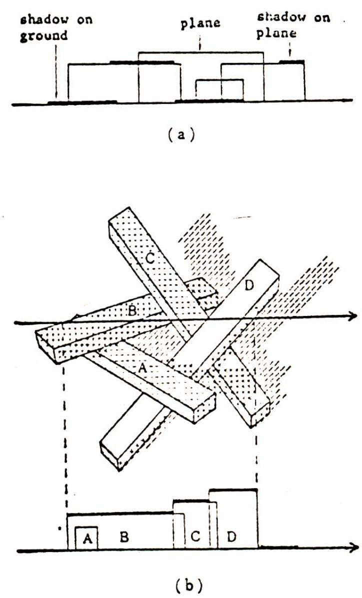


Fig.5 Relation of objects on scanning line.

The outline of the program based on the algorithm mentioned above is shown in fig. 6, where n_o is the number of convex polyhedra, k=1 gives the flow for a light source and k=2 does that for an observation point.

3. Examples and conclusion

The examples obtained by employing the algorithm

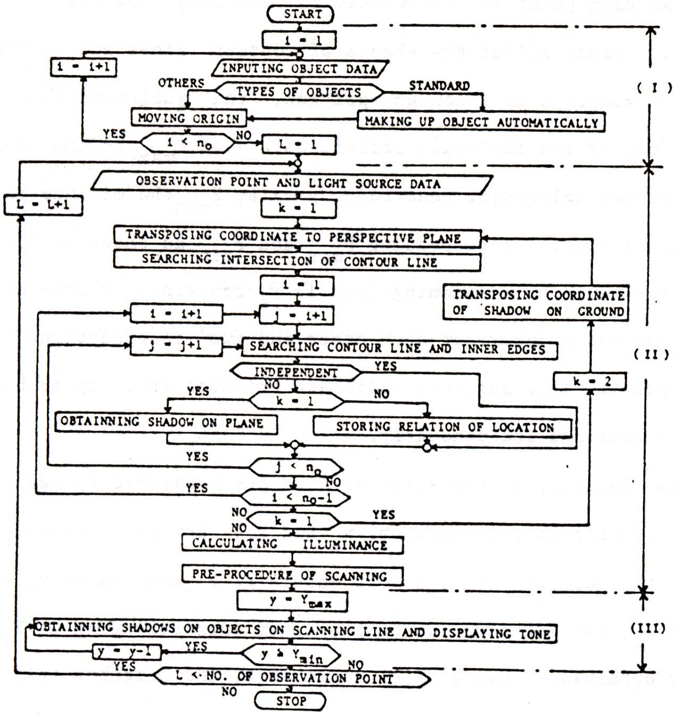


Fig.6 Flow-chart.

explained in this paper are shown in fig.7. The figures,(a) and (b), are the examples composed of only the set of the standard convex polyhedra prepared in the program, and (a), (b) and (c) are displayed by a line printer. The number of meshes is 135×180 and the number of tone steps is 20. The figures, (d), (e) and (f), are displayed by a XY-

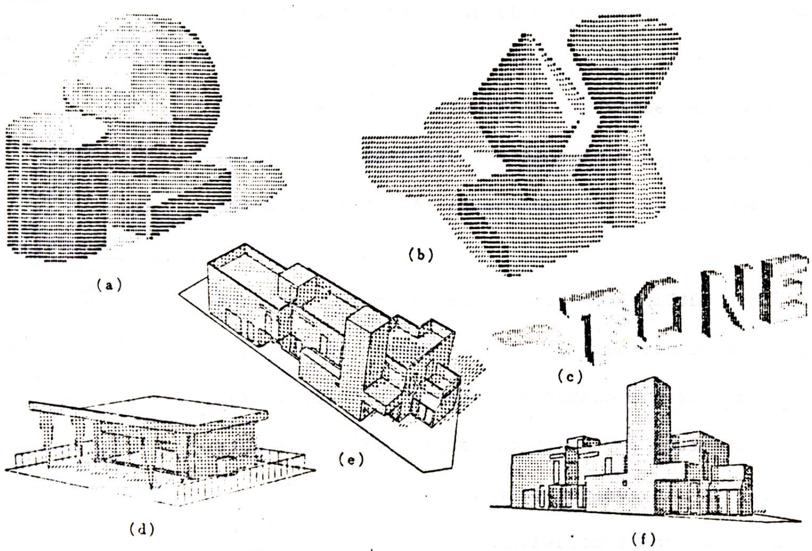


Fig.7 Examples.

NO. OF FIGURES	NO. OF CONVEX POLYHEDRA	NO. OF VERTICES	NO. OF PLANES	CPU-TIME (sec.)			
				I	II	III*	ELIMINATING HIDDEN LINE
(a)	3	122	118	0.19	0.48	0.46(7.41)	
(b)	5	120	70	0.30	0.80	0.33(7.72)	
(c)	13	104	78	0.62	0.63	0.31(7.59)	
(d)	15	76	50	0.65	0.80	0.44(2.55)	0.78
(e)	27	218	163	1.14	2.80	0.99(6.15)	1.75
(f)	27	218	163	1.14	2.69	0.59(5.40)	1.52

* The numbers of parentheses show the print instruction times.

Table 1, Computing time.

recorder to show the boundary lines of planes which have same illuminance and to treat the figures on planes and the lines in space. In these cases, the number of tone steps is 10.

The hidden line algorithm in the reference [4] was applied. The program is written by FORTRAN IV. The computing times on FACOM 230-60 are shown in Table 1, where the computing times in the table correspond to fig.7.

The equations used in this program are very simple. Furthermore, the values once obtained can be used repeatedly for many purposes so that the computing time becomes shorter exceedingly. By means of the pre-procedures applied the feature of convex polyhedra, the scanning time is reduced and is not very much affected by the number of meshes of an output picture.

Reference

[1] P.P. Loutrel: A solution to the hidden-line problem for computing drawn polyhedra IEEE Trans. Computer, C-19 (March 1970).

[2] F.J.Mcgrath: A method for eliminating hidden lines with polyhedra, Simulation, p. 37 (Jan. 1971).

[3] C.B.Jones: A new approach to the hidden line problem, The Computer Journal, 14, 4,p.232.

[4] Nakamae, Nishita: An algorithm for hidden line elimination of polyhedra, Information processing society of Japan, vol.12,p.134 (1972).

[5] C.Wylie, G.Romney, D.Evans, R.Erdahl: Half-tone perspective drawings by conputer, Fall joint computer conference, (1967).

[6] A.Appel: On caluculating the illusion of reality, Hardware, p.945 (1969).

[7] G.W.Romney, G.S.Watkins, D.C.Evans: Real time display of computer generated half-tone perspective pictures, Hardware, p.973 (1969).

[8] W.J.Boknight: A procedure for generation of three dimensional half toned computer graphics presentation, Comm.ACM, 13, 9, (1970).

[9] R.Goldstein, R.Nagel: 3-D visual simulation, Simulation, p.25 (1971).

[10] H.Gouraud: Continuous shading of curved surfaces,IEEE Trans.Computers,p.623(June 1971).