CONTINUOUS TONE REPRESENTATION OF THREE-DIMENSIONAL OBJECTS ILLUMINATED BY SKY LIGHT

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Abstract

Natural lighting models to date have been limited to calculation of direct sunlight. However, this paper proposes an improved model for natural lighting calculations that adequately considers both direct sunlight and scattered light caused by clouds and other forms of water vapor in the air. Such indirect natural light is termed skylight and can be an important factor when attempting to render realistic looking images as they might appear under overcast skies.

In the proposed natural lighting model, the sky is considered to be a hemisphere with a large radius (called the sky dome) that acts as a source of diffuse light with nonuniform intensity. In order to adequately take into account the nonuniform intensity of such skylight, the sky dome is subdivided into bands. The light intensity within individual bands can be assumed to be transversely uniform and longitudinally nonuniform and therefore the total luminance emanating from each band can be calculated more accurately.

The proposed method significantly improves the realism of natural lighting effects. Its advantages are particularly apparent when simulating lighting under an overcast sky or when rendering surfaces that fall within a shadow cast by an obstruction lit by direct sunlight.

CR Categories and Subject Descriptors: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism; I.3.3 [Computer Graphics]: Picture/Image Generation

General Terms : Algorithms

Additional Key Words and Phrases: shading, sky light, shadows, interreflection of light, daylight factor, distributed light source

1. INTRODUCTION

Recently, various display techniques have been developed by numerous researchers, for such things as trees, natural terrain, clouds and fog effects [1-5]. Still other algorithms have been developed to treat such problems as texture mapping, reflection, refraction and transparency [6,7]. Much of this research has been designed to facilitate generation of more and more realistic looking simulations of various environments.

The visual appearance of such simulated environments depends to a great extent on the lighting model used to illuminate the objects in the environment. In this connection, researchers have already developed lighting models for various types of light sources [8-9] as well as algorithms that take into account the effect of interreflection among objects [10-12].

However, to date, lighting models have been limited to either artificial light sources or direct sunlight. In fact, though, a high percentage of the environments to be simulated are naturally lit by what is commonly called daylight; i.e. a combination of direct sunlight and skylight. One of the most troublesome problems when using only direct sunlight as a light source is that objects in shadow appear to be unnaturally dark. One common solution to this problem is to add a uniform ambient light to the lighting model. But this tends to hide any surface unenvenness of the objects in shadow since all parts of the surface are lit evenly. Another common approach is to add a secondary light source coincident with the viewer's eye point. While this is a convenient solution, it detracts from the realism of shaded renderings. In order to overcome such problems in a general way, this paper proposes a lighting model for skylight which, when combined with direct sunlight, allows increased realism when rendering natural environments.

Fundamentally, skylight can be thought to emanate from the sky dome which surrounds the earth. The sky dome can be treated as a hemisphere of very large radius. However, skylight intensity is not uniform accross the entire surface of the sky dome and will also vary greatly with the weather (e.g., clear sky or overcast sky). In order to handle such nonuniformity in a consistent manner, the paper introduces the idea of subdividing the sky dome into light emanating bands whose intensity is uniform within its boundaries; the idea that the intensity of an entire band can be approximated by calculating the intensity of sample lines located at the center of such bands; and the idea of alternative methods for calculating such intensities under either clear or overcast skies.

2. ASSUMPTIONS AND FUNDAMENTAL IDEAS

As already mentioned, daylight is a combination of direct sunlight and diffuse skylight. This paper will only discuss how to handle skylight. Direct sunlight can be handled in the normal way, which is to treat it as a parallel light source.

Intensity of skylight (also called sky luminance) is not uniform and its distribution varies significantly depending on whether the sky is clear or overcast. Such variations due to weather conditions are discussed in this paper.

The color of skylight is also not uniform and its spectral distribution varies significantly depending on solar altitude. This is most obvious at sunrise and sunset when the sun is low on the horizon. Although some of the most beautiful natural lighting effects occur at these times of the day, the issue of spectral distribution is not discussed in this paper. However, spectral distribution is suggested as topic for future research within the conceptual framework of the model for skylighting proposed here.

Before beginning the calculation of illuminance at a given point in the scene, it is necessary to test for obstructions and the resulting shadows. The shadow boundaries that would be caused by direct sunlight on a clear day are determined by an overlap test of objects on the perspective plane when viewed from the sun (see [13]).

The illuminance from skylight can be calculated by determining the area of the sky visible from the given point in the scene. In the proposed lighting model, the skydome is divided into bands which themselves do not necessarily have uniform luminance. Therefore, the total luminance from a visible area of the sky must be derived by summing the luminances of the visible areas of the individual bands.

The illuminance at a given point in the scene is a combination of light from the sky and light reflected from the ground and/or surrounding objects. The ratio between skylight and reflected light from the ground depends on the declination of the plane including the given point in the scene.

Interreflection of light between objects in the scene must also be considered in order to acheive an accurate simulation of natural lighting. This is particularly important when creating interior scenes with many objects in a room. In such cases, the interreflection luminance can be determined by first subdividing the faces of the objects in the scene (such as walls, furniture, etc.) into a suitable number of subsurfaces and then calculating the interreflection at the vertices of each subsurface (see [11]).

The ground and all other objects in the scene are assumed to be sets of convex polyhedra and convex polygons. The surfaces of all polyhedra are assumed to be Lambertian surfaces; i.e., perfectly diffuse. The ground itself is treated as a large area surface with uniform luminance.

3. ILLUMINANCE CALCULATION FOR SKY LIGHT

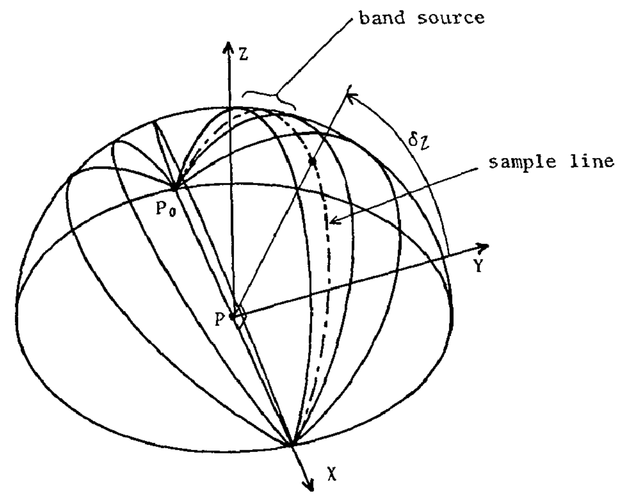


Fig. 1 Subdivision of the sky dome to band sources (P:the calculation point).

If the intensity of sky light were uniform; it has been shown that the illuminance at a given point can be calculated by using a configuration factor. A configuration factor is the ratio of the area of the visible sky projected onto the base of the sky dome vs. the total area of the base of the sky dome(This ratio is also called the sky factor). Illuminance at a given point is then obtained by the multiplication of the configuration factor and the sky luminance. The configuration factor is usually obtained by contour integration of the visible areas of the sky (refer to Ref. [9] for the contour integration method for area sources).

However, the distribution of sky luminance is nonuniform in most cases. The distribution of the sky luminance varies depending on sites, time, and seasons. In order to deal with varied distributions, the following methods might be considered:

(a) Generate many sample points on the sky dome, and set the weighting factor proportional to sky luminance of each point.
(b) Subdivide the sky dome into many finite elements (e.g., rectangular elements).
(c) Subdivide the sky dome into many band elements (we call these elements "band sources").

In methods (a) or (b), both the illuminance calculation and the visibility test used to calculate the effect taking account of obstructions have to be done for each point or each element, respectively. Method (c) is less costly compared with the other methods for the following reasons (see Fig.1).

(1) The calculation point can always be handled as the effective center of the sky dome hemisphere because of the dome's very large radius. (2) The luminance of band sources is assumed to be continuously varying with respect to the longitudinal direction of the band source. (3) The center line of the band source is called a "sample line" and a plane consisting of the sample line and the center of hemisphere is called a "sample plane". The sample plane of the ℓth band source, S_ℓ, is defined as the intersection line between the hemisphere and the plane whose tilt angle is δ_ℓ. The visible parts of the band sources are determined by using these sample lines. Then the illuminance can be obtained by adding the

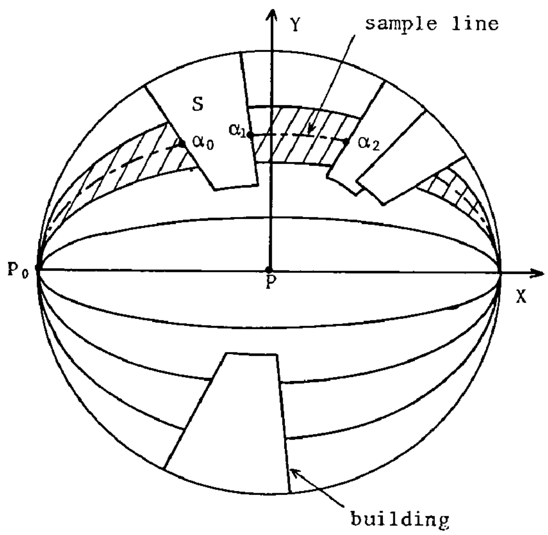


Fig. 2 Calculation of visible areas of a band source by using a sample line.

illuminances of the visible parts of the band sources (Fig.2 shows the visible parts of sample lines on the orthogonal projection); the intersections between obstacles and sample lines can be obtained by testing the intersections between the obstacles and the sample planes in the object space (see 3.3).

3.1 Illuminance Calculation from only Sky Light

The calculation method of illuminance on skyward-facing horizontal surfaces, which are top surfaces of objects such as the ground, is discussed here. In this case, taking reflected light from the ground into account is unnecessary.

3.1.1 Illuminance from Unobstructed Sky

As mentioned before, the calculation point can always be handled as the center of the sky dome. For simple calculation, we define the coordinate system so that the horizontal plane including the calculation point is the X-Y plane, which is coincident with the base of the sky dome with a radius r; the calculation point is the origin of the coordinate system.

The sample planes are determined as declined planes which include the X-axis and whose elevation angles are δ_ℓ ($\ell=1,2,\ldots,N$; N=the number of subdivisions) (see Fig.3). The band source may be broken down into small "sky elements", each of which may be considered to have a constant luminance. By treating the sky elements as a point source P_e, the inverse-square law may be applied for illuminance at a calculation point P. By expressing the position of P_e with angle α from X-axis (PP_0) to sky element and angle δ from horizontal plane to sky element, the illuminance at P due to the sky element represented by P_e is given by

$$dE = L(\alpha,\delta)\cos\theta/r^2 dA \ , \qquad (1)$$

where L is the sky luminance at P_e, θ is the angle from the zenith to P_e ($\cos\theta = \sin\alpha\sin\delta$), r corresponds to the distance between P and Pe, and

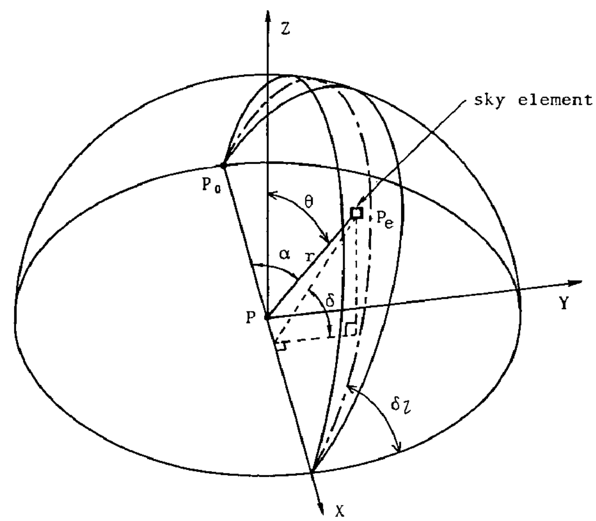


Fig. 3 Distribution of sky luminance.

dA is the area of the sky element ($dA=(rd\alpha)\cdot(rd\delta\sin\alpha)$) (see Fig.3). By integrating equation (1) with respect to α and δ, the illuminance due to the band source between 0 and α is given by

$$E_\ell(\alpha) = \int_0^\alpha\int_{\delta'_\ell}^{\delta'_{\ell+1}} L(\alpha,\delta)\ \sin\delta\sin\alpha^2 d\delta d\alpha \quad , \qquad (2)$$

where $\delta'_\ell = \delta_\ell - \Delta\delta$, $\delta'_{\ell+1} = \delta_\ell + \Delta\delta$, $2\Delta\delta$ is the angular width of a band source.

Let's consider the case of a uniform sky luminance first. In this case, $L(\alpha,\delta)=L_0$ (i.e., constant), then equation (2) becomes the following equation.

$$E_\ell(\alpha) = 0.5 d_\ell(\alpha - \cos\alpha\sin\alpha)L_0 \ , \qquad (3)$$

where $d_\ell = (\cos\delta'_\ell - \cos\delta'_{\ell+1})$. In this paper, we set the angle of the sample plane, δ_ℓ ($\ell=1,2,\ldots,N$), as $d_\ell = 1/N$ (constant).

In the case of nonuniform sky luminance, if it is assumed that the luminance in the small width, $\delta'_\ell < \delta < \delta'_{\ell+1}$, is constant, then $L(\alpha,\delta)$ is represented by $L(\alpha,\delta_\ell)$. Thus, equation (2) becomes the following equation.

$$E_\ell(\alpha) = d_\ell\int_0^\alpha L(\alpha,\delta_\ell)\sin^2\alpha\ d\alpha \ . \qquad (4)$$

Empirical formulas for calculating the luminance of both overcast skies and clear skies have been determined by the CIE for international use; the distribution is defined as the relative luminance to the luminance of the zenith.

a) the overcast sky (the CIE Standard sky luminance function[14])

$$L(\theta) = Lz(1+2\cos\theta)/3 \quad , \qquad (5)$$

Lz: luminance of the zenith,
θ : the angle from the zenith to a sky element,

where Lz is the function of the solar altitude, but $L(\theta)$ is independent to the sun position. The above equation means that the sky luminance is the highest at the zenith and the lowest around the horizon.

b) the clear sky (the CIE Standard sky luminance function[15])

$$L(\theta,\phi)=L_z\frac{(.91+10\exp(-3\gamma)+.45\cos^2\gamma)(1-\exp(-.32\sec\theta))}{0.274(0.91+10\exp(-3z_0)+0.45\cos^2 z_0)} \quad (6)$$

γ : the angle from the sun to a sky element; ($\cos\gamma=\cos z_0\cdot\cos\theta+\sin z_0\cdot\sin\theta\cdot\cos(\phi-\phi_0)$)
z_0: the angle from the zenith to the sun,
ϕ : the azimuth angle from X-axis to the sky element,
ϕ_0: the azimuth angle from X-axis to the sun.

The above mentioned equation means that the sky luminance varies with the sun position. Sky luminance is highest in the region of the sun and lowest at about 90° from it.

3.1.2 Illuminance Calculation Taking Account of Obstructions

$E_\ell(\alpha)$ in equation (2) is the illuminance due to the region between 0 and α in a band source. Therefore, if the region between α_1 and α_2 , where $\alpha_1\leq\alpha_2$, is visible from the calculation point, P (see Fig.2), the illuminance by the source in this region can be easily obtained as $E_\ell(\alpha_2)-E_\ell(\alpha_1)$. The visible parts detection method is described in the section 3.3 of this paper. It is convenient to precalculate values of $E_\ell(\alpha)$ and store them in look up tables. In this paper, values are prepared every ten degrees, and the E_ℓ at an arbitrary α is computed by linear interpolation.

In case such as a partly cloudy sky which fall between a completely overcast sky and a completely clear sky, we can blend the results of equations (5) and (6) even though the approximation precision is not very accurate because of the varied distribution of clouds. Using a distribution of measured data would be better. Look up tables could be obtained for any particular kind of sky conditions if the distribution was given. And if the measured data included spectral distributions, it would be possible to compute illuminance taking into account spectral variations by preparing look up tables as functions of the spectrum.

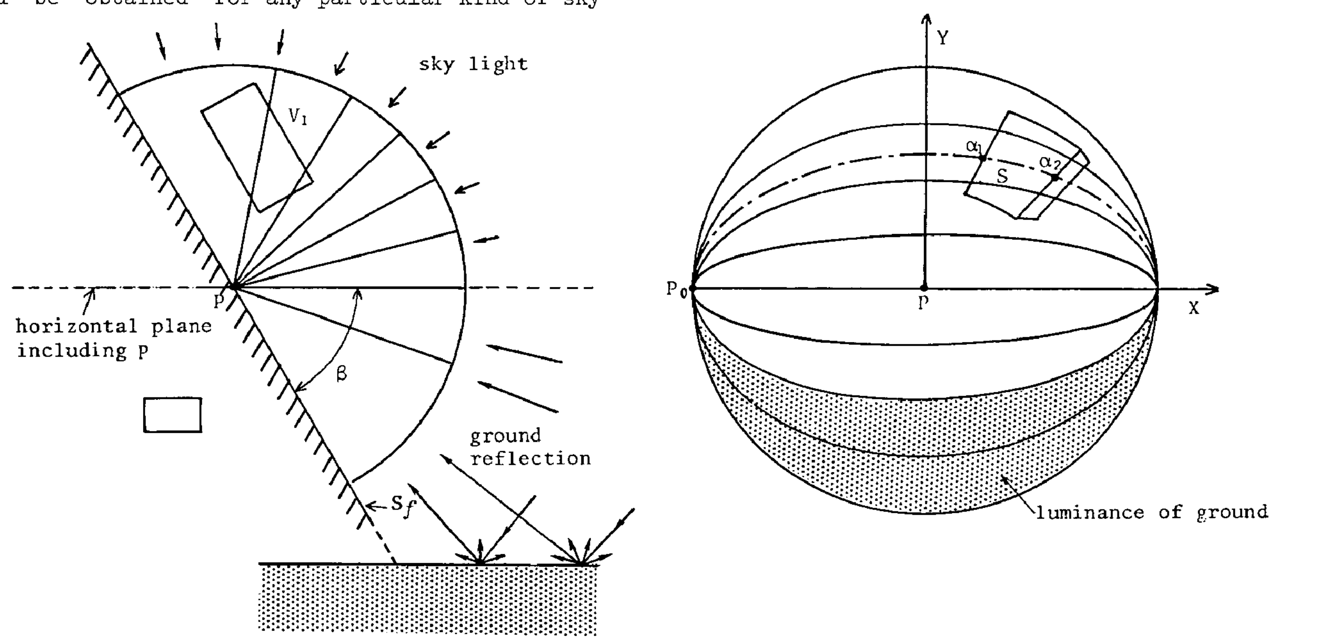
3.2 Illuminance Calculation Taking into Account Reflected Light

Reflected light is composed of light reflected from the ground, light reflected from buildings, and the interreflection of light between buildings, ground and other objects (see Ref.[11,12] for interreflection of light). First, we describe the reflected light from the ground.

Sloped surfaces are usually illuminated by reflected light from the ground. Assuming that the ground is a large surface with a uniform brightness, the illuminance on the sloped surfaces can be obtained by inclining the hemisphere (see Fig.4-a). That is, the upper part of the hemisphere above the horizontal plane including the calculation point has the sky luminance, and the lower part has the luminance of the ground. The illuminance of the ground is obtained by the product of the sky illuminance and the ground reflectance, where the illuminance on the ground is assumed to be the illuminance from the unobstructed sky.

When the inclined angle of the face S_f is β, the illuminance on S_f can be calculated with a look up table, $E_{\ell\beta}(\alpha)$ which is pre-calculated by inclining $L(\alpha,\delta)$ by the angle β. The part of $L(\alpha,\delta)$ under the horizontal plane is set as the luminance of the ground (the dotted area in Fig.4-b shows the luminance of the ground).

In order to simplify the calculation, we assume that the illuminance of each face is uniform (the area, which can't be assumed uniform, may be subdivided into subsurfaces). Let's consider the reflected light from a face S in Fig. 4-b, the face S exists between α_1 and α_2 on the sample line, then



(a):side view (b):projection onto the base of the hemisphere

Fig. 4 Illuminance calculation for a sloped plane
(the origin of the coordinate system is the calculation point P; the X-axis is parallel to the horizontal plane)

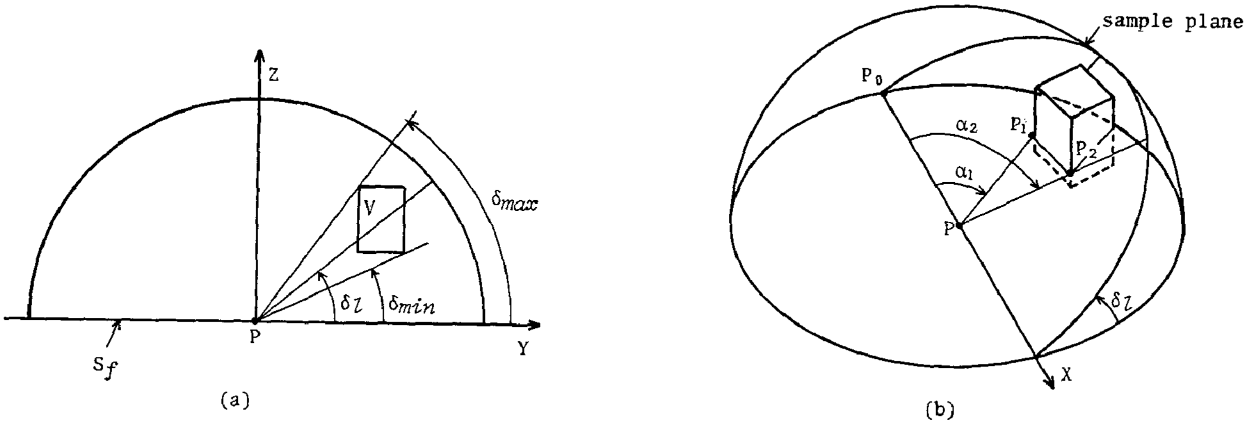


Fig. 5 Intersection test between sample planes and objects.

the illuminance due to this region can be obtained by substituting Lo in equation (2) to the luminance of S. This calculation is done only for faces visible from the calculation point, and can be omitted for small and low reflectance faces.

3.3 Calculation of the Visible Parts of the Sky

In order to consider the effect of obstructions, the visible parts of the sky viewed from the calculation point must be determined. The illuminance at a point P on a face S_f can be obtained by extracting the visible sections of the band sources; the procedure is as follows:

1) Extract the objects that cast shadows on the face S_f (i.e., objects existing in front of S_f; e.g., V_1 in Fig.4-a).
2) Transform the coordinates of these objects in order to set S_f as the base of hemisphere (see Fig.5-a).
3) Calculate the existing range of each object (the range is defined by the elevation angle; e.g., δ_{min} and δ_{max} in Fig.5-a).
4) Extract the contour lines of each object when viewed from the calculation point.
5) Execute the following calculation sequence for each sample plane;
   i) Extract the objects intersecting with the sample plane by using the range solved in 3) and the elevation angle of the sample plane.
   ii) Calculate the intersections between the contour line of each object and the sample plane, and get the angles, α_i (i=1,2,..,m), corresponding to the intersections (in Fig.5-b, α_1 is obtained by the inner product of vectors PP_1 and PP_0).
   iii) Extract the visible sections of the sample line (e.g., the section $\alpha_1\alpha_2$ in Fig.2, see [16] for hidden line elimination).
   iv) Calculate the illuminance due to each visible section by using the look up table, E_ℓ.

3.4 Illuminance Calculation for Naturally Lit Interiors

The illuminance at a calculation point in a room is usually influenced not only by the sky light and the ground light coming in through the windows but also by interreflection of light between objects in the room. Therefore, in the first step, the illuminance due to the light source from the windows is calculated, and then the interreflection is calculated by using the direct illuminance from windows (see Ref.[11] for the calculation of interreflection).

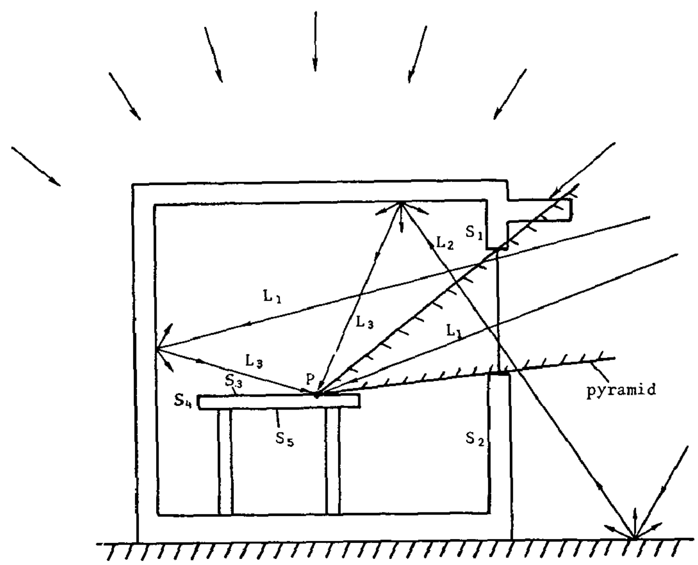


Fig. 6 Components of light in a room.
(L_1:direct light from sky, L_2:reflected light from the ground, L_3:interreflection of light)

It is unnecessary to consider the whole sky for the shading calculation of an interior because of the limited regions of the windows.

Therefore, for the calculation of the direct component from the sky light, the following terms should be considered for interiors :

a) The illuminance on a face, from which all windows are invisible, is zero (e.g., the window is on the back side of the faces such as S_1, S_2, S_4, S_5 in Fig.6).
b) Sample planes are only required to be generated within the range of the windows.
c) For shadow calculations, it is enough to extract the objects only within the pyramids composed by the windows and the calculation point (see Fig.6).

4. DAYLIGHT FACTORS

The illuminance from windows depends on the sky luminance. Therefore, the daylight factor is usually used for lighting design in daytime: it is defined as the ratio between the daylight illuminance at a point in the interior and the simultaneous exterior illuminance available on a horizontal plane from an unobstructed sky expressed as a percentage.

Superimposing the color belts of the daylight factor on the shaded images is used in the examples here; each color corresponds to the value of its daylight factor (see examples in Fig.10). With this depiction method, we can easily grasp the numerical values and the scene simultaneously.

5. EXAMPLES

Fig.7 shows the shaded images of a simple object. Picture (a) shows the object with only direct sunlight, (b): *uniform* skylight, (c): *an* overcast sky, (d): a clear sky (only skylight), and (e): a clear sky (including direct sunlight).

Fig.8 shows the outdoor scenes of a building under various conditions. Picture (a) shows the building with direct sunlight and uniform ambient light. Pictures (b) and (c) are with an overcast sky and a clear sky, respectively. Pictures in this sequence take into account reflected light from the ground and other objects.

In the previous methods, shadows under an overcast sky are ignored. As shown in Fig.8-b and c, there are penumbrae which means that the boundaries of shadows are not sharp. These penumbrae are more realistic and make it easy to recognize the shapes of objects in shadows. In Fig. 8-a and Fig.7-a, it is difficult to grasp the shapes of objects in the shadows because only direct sunlight and uniform ambient light are considered.

Fig.9 shows the interior of a computer room illuminated by a clear sky; in this figure the interreflection of light from walls and objects is taken into account. Notice that the parts of the walles farther from the window are also bright. (All sample images found in this paper were made in this room.)

Fig.10 shows examples of daylight factors. Pictures (a) and (b) show the distribution of daylight factors with an overcast sky and with a clear sky, respectively; the daylight factor is defined only for sky light, then (b) does not include direct sunlight. Picture (c) shows the distribution of daylight factors with a clear sky. The color belts on the images show that the distribution of illuminance caused by the sky light is very complex and depends greatly on the sun's position. However, using the proposed method, we can design the daylighting of the room accurately.

Table 1 shows the computation times for Fig.8(b) and Fig.9. All computation was performed on a TOSBAC DS-600/80 32bit minicomputer. In this paper, the multi-scanning method is used for anti-aliasing (see section 2.5 in Ref.[17]). In these examples, Mach bands are slightly visible, because the illuminance at the calculation point is sampled along the sample lines.

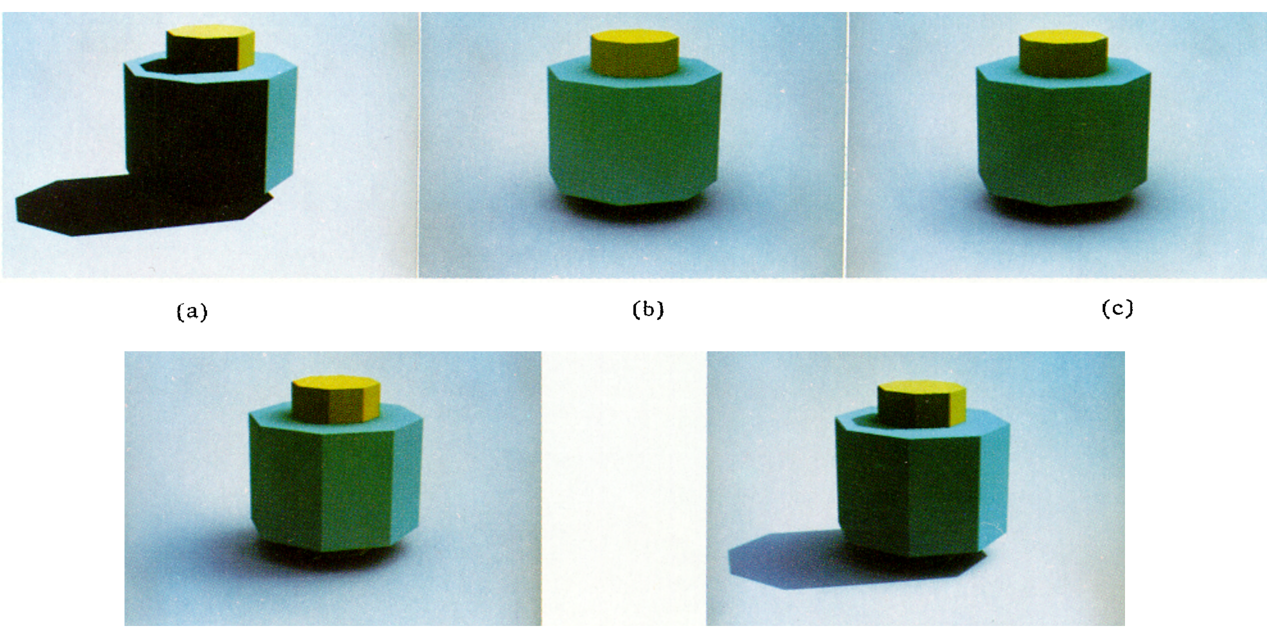
In these examples, the ground reflectance is 0.25, and the number of sample planes (see section 3) for Fig.8 and Fig.9 are 20 and 200, respectively. Note that the effective sample planes for Fig.9 are limited (see 3.4).

These examples make clear that accounting for sky light and its shadow effects is very useful for generating realistic images, especially in cloudy weather, and is helpful for designing buildings and lighting interiors.

6. CONCLUSION

A representation method for three-dimensional objects illuminated by sky light has been proposed.

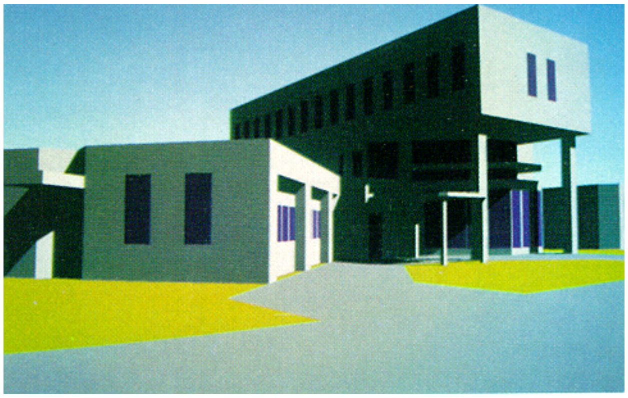
The following conclusions can be stated from the



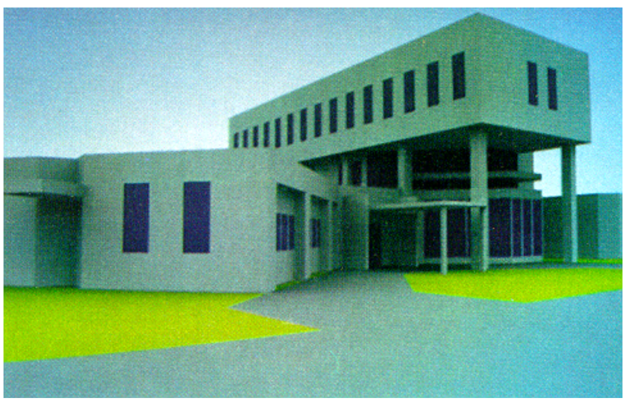
(a) (b) (c)

(d) (e)

Fig. 7 Example for a simple shaped object.
a:direct sunlight, b:uniform skylight, c:overcast sky,
d:clear sky, e:clear sky(including direct sunlight)



(a)

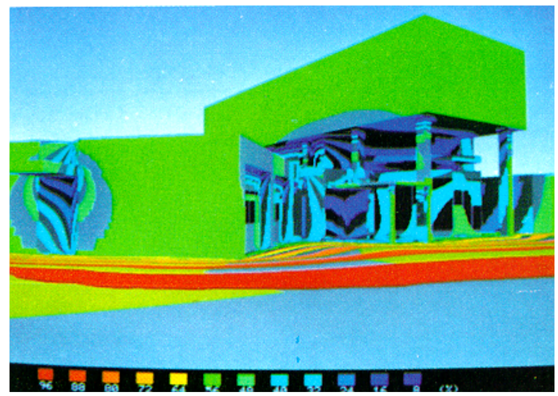


(b)

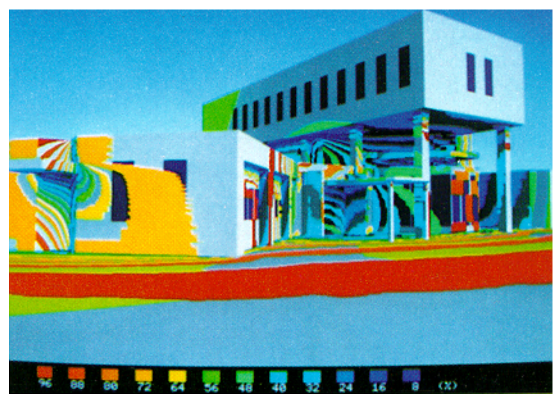
(c)

Fig. 8 Examples for exterior scene.
a: direct sunlight, b: overcast sky
c: clear sky (including direct sunlight)

Fig. 9 Example for interior scene.



(a)



(b)

(c)

Fig. 10 Examples for daylight factors.
(a:overcast sky, b:clear sky, c:clear sky)

Table 1. Computation times

	number of faces	image size	modeling and hidden-surface removal(min.)	shading(min.) direct illumination	interreflection of light
Fig.8(b)	239	480x317	1.56	27.67	6.90*
Fig.9	231	480x325	1.81	20.91	16.08

*only the 1st order of reflection is considered.

results.

(1) Images are much realistic using sky light and its reflections. The proposed method especially improves the realism of buildings under both overcast skies and clear skies, as well as interiors illuminated by sky light through windows.

(2) The illuminance from sky light can be obtained by considering the sky as a hemisphere with a large radius. Subdividing the hemisphere into band sources allows us to compute the illuminance due to nonuniform sky luminance.

(3) The effect of sky light obstructed by objects can be computed by summing the illuminances due to the visible parts of each band source viewed from the calculation point; the illuminance for each visible part can be easily calculated by a look up table for integrated illuminance.

(4) The effect of light reflected from the ground to sloped surfaces can be computed by the ground luminance for the under part of the horizontal plane including the calculation point.

(5) Displaying daylight factor considering the distribution of sky luminance is useful for lighting design.

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