Method of Displaying Optical Effects within Water using Accumulation Buffer

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Abstract

A precise shading model is required to display realistic images. Recently research on global illumination has been widespread. In global illumination, problems of diffuse reflection have been solved fairly well, but some optical problems after specular reflection and refraction still remain. Some natural phenomena stand out in reflected/refracted light from the wave surface of water. Refracted light from water surface converges and diverges, and creates shafts of light due to scattered light from particles. The color of the water is influenced by scattering/absorption effects of water molecules and suspensions. For these effects, the intensity and direction of incident light to particles plays an important role, and it is difficult to calculate them in conventional ray-tracing because light refracts when passing through waves. Therefore, the pre-processing tracing from light sources is necessary.

The method proposed here can effectively calculate optical effects, shafts of light, caustics, and color of the water without such pre-processing by using a scanline Z-buffer and accumulation buffer.

CR Categories and Subject Descriptors:
I.3.3 [**Computer Graphics**]: Picture/Image Generation
I.3.7 [**Computer Graphics**]: Three-Dimensional Graphics and Realism
Key Words: Shaft of light, Caustic, Color of Water, accumulation buffer, Optical Length, Photo-realism

1 INTRODUCTION

Image synthesis of natural phenomena is one of the most popular field these days; displays of mountains, trees, the earth, waves, the atmosphere have been attempted. This paper is concerned with optical effects within water: the proposed method calculates and displays shafts of light, caustics due to refracted light passing through waves, and the color of water.

Creatures in water and illumination effects onto them are spectacular; Such images are often used in commercials these days, though they are not physically-based images.

In the discussion of optical effects under water, the following three factors must be carefully taken into account; shafts of light and caustics due to refracted light from waves and the color of water due to scattering/absorption of particles.

Concerning caustics due to waves, Shinya [21] precalculated illumination distribution on surfaces by using *grid-pencil tracing*. Watt [22] developed *backward beam tracing*. These methods are 2 pass solutions and require storage memory for an illumination map or caustic polygons. The backward beam tracing method is the improved version of the beam tracing method [6] which can handle reflection/refraction light for polygonal objects. Therefore the method can only handle polygons. Heckbert [7] has developed a method which can calculate caustics due to refracted light from transparent objects such as lenses; the illumination distribution on surfaces is stored as textures. Chen [3] calculates an illumination map by using the *multi-pass method*. But these methods using maps have difficulties in resolution and aliasing (artifact). Mitchell [13] calculated caustics from curved reflectors (limited to implicit surfaces) by using the numerical methods. However, none of these methods have discussed shafts of light.

Concerning shafts of light, Blinn [1], Max [12], Nishita [16], and Klassen [11] have all developed methods for atmospheric scattering. Watt [22], as mentioned above, realized the display of shafts of light underwater by using Nishita's method [16] based on ray-tracing.

Concerning the display of a water surface, Max [12], Fournier [4] and Ts'o [20] have all made an attempt, but they didn't take into account scattering effects due to particles in the water, instead discussing shapes of waves. The authors have developed calculation methods for the color of water surface such as ponds [9] and the color of the sea as viewed from outer space [18]. None of them have discussed the color when viewed from a point within the water.

For displaying creatures in the water, free-form surfaces are indispensable, and the display of caustics on such curved surfaces is desired. The method proposed here can handle free-form surfaces, and is a combined method of scanline based Z-buffer, shadow volume and A-buffer [5]; the method can calculate shafts of light and caustics without the ray-tracing methods used in previous methods. In our method, as a side effect of calculation of shaft of light, caustics on objects can be obtained; in previous work, the shafts of light were calculated by the integration of scattered light on a viewing ray at each pixel, whereas in this method the intensity at every pixel on each scanline is obtained by scanning the intensities passing through the scan plane by the following reasoning. The particle density (mainly water molecules) in the water is assumed as uniform, so the intensity due to scattered light can be obtained as an analytical function of distance between the particles and the viewpoint.

In this paper, to represent curved surfaces the improved method [19] of *metaball* [14] (or referred as blobs [2]) is used.

2 PROPOSED MODEL

The main purpose in the proposed model is to display the following optical effects; (a) shafts of light due to the convergence/diversion depending on wave shape, and (b) caustics on curved surfaces. This model takes into account scattering/absorption due to particles (water molecules and suspensions) in the water, and assumes uniform intensity distribution within visible regions and ignore multiple scattering. For light sources, direct sunlight and skylight outdoor, and point light sources and ambient light indoor are taken into ac-

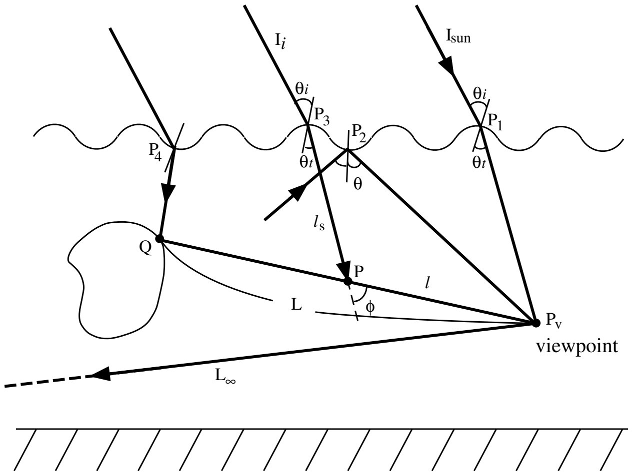

Figure 1: The optical paths arriving at the viewpoint.

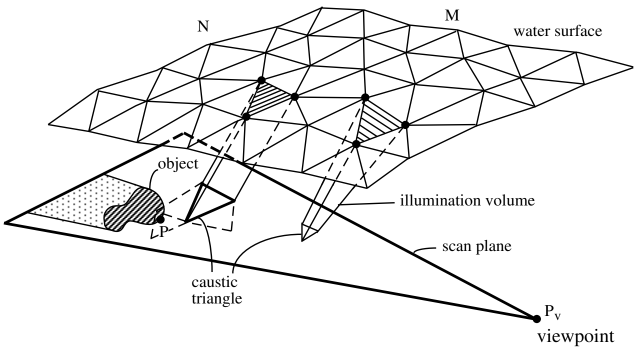

Figure 2: Geometry of illumination volumes and caustic triangles on a scan plane.

count, even though only the former is discussed in this paper. As our topic is optical effects within water, shadows caused by objects above water surfaces are ignored.

2.1 Optical-path model

As shown in Fig.1, when one's viewpoint is located within the water, we can categorize the optical paths arriving at the viewpoint into the following three.

(a) Viewing ray intersecting with objects: The refracted light after passing through the water surface is reflected by an object (point Q in Fig.1), and arrive at the viewpoint. The light is scattered and attenuated by particles (e.g., P in Fig.1) on its way.

(b) Viewing ray intersecting with water surface: The refracted light after passing through the water surface, incoming light (direct sunlight and skylight) arrives at the viewpoint directly. The optical paths are classified into the following depending on the angle between the normal of the wave surface and the viewing ray.
(b.1) the angle is less than critical angle (P_1 in Fig.1): the component of refracted light in the incident light at a point on the water surface arrive at the viewpoint.
(b.2) the angle is larger than critical angle (P_2 in Fig.1): the light which perfectly reflects at a point on the water surface arrives at the viewpoint.

(c) Viewing ray having no intersection (L_∞ in Fig.1): As the viewpoint is located within the water, the ray usually intersects with something such as the bottom of water. Strictly, the light path belongs to either case (a) or (b). As the extinction coefficients of particles in water is high in general [8], the light from distant intersection points can be ignored.

2.2 Basic ideas

Concerning light refracted after passing through waves, the following conditions should be considered. (i) The intensity distribution of the light in space is not uniform because refracted light converges and/or diverges. (ii) The intensity of reflected light from objects consisting of two component, diffuse and specular reflection, depends on the incident direction of light. (iii) The intensity of scattered light from particles strongly depends on the phase angle between the incident direction and the scattering direction; this effect is called phase function.

It is very difficult to know the path of light arriving at an object (Q in Fig.1) and a particle (P in Fig.1) and the intensity distribution due to caustic effect because of refraction. To solve this problem, previous methods (e.g., [22]) employ an illumination map, whereas our method does not need such a map, as is described below.

Let's consider subdivided triangle elements from a lattice after meshing a water surface (see Fig.2). The refraction vectors are calculated at every lattice point and swept, and a swept volume of a triangle along the refracted vectors is called the *illumination volume*.

For visualization of transparent objects with non-uniform density such as clouds, smoke or haze, numerical integration of the densities on the ray is required. However, this is unnecessary as the density of particles in water is uniform. In the proposed model, for the calculation of the intensity distribution of light in a uniform density medium, that is in water, an analytical function can be used. Our method can be applied not only to water but also to an atmosphere with uniform density.

The calculation of shafts of light and caustics is equivalent to calculating the intensity distribution of flux of light in space. It is difficult that the space to be calculated is limited to the visible regions within the field of view, which is filled with water. To obtain this space, the Z-buffer method can be used. The visible space within the water at each pixel can be obtained as the front part of the depth of the objects stored in the Z-buffer. That is, the invisible regions on a scan plane (e.g., the dotted region in Fig.2) can be discarded.

The intensity of light is inversely propotional to the intersection area between scan plane and illumination volume. Then, the intensities of scattered light due to particles can be obtained by applying scan conversion on the illumination volumes. That is, after calculation of intensity distribution on a scan line, these intensities are stored in an accumulation buffer.

In this paper, metaballs are employed for displaying curved surfaces. In the method used here, the intersection test between the viewing ray and iso-potential surface can be done through ray-tracing, in which it is effectively executed because the density function on the ray is represented by degree six Bézier curves. The intersection test is performed by *Bézier clipping* [17].

2.3 Optical model

In water the light arriving at the viewpoint has two components; the light arriving at the viewpoint directly from light refracted after passing through waves, and the indirect light arriving from light reflected from objects and from scattered light from particles. For the latter, the calculation is done by taking into account light distribution within water. The following conditions also need to be taken into account.

(a) Even though the distribution of light within water is determined by the incident light, direct sunlight and skylight over the water onto wave surfaces, the shaft of light can be regarded as being generated only by strong direct sunlight; i.e., skylight can be neglected. In our model the illumination volumes caused by refracted light consist of only direct sunlight.

(b) The scattered light from outside of the viewing field can't arrive at the viewpoint because multiple scattering due to particles is ignored. Light from distant points can't arrive at the viewpoint because the extinction coefficient of particles within the water is large. Defining the maximum distance of light arriving as visible length, the space to be calculated is finite; the viewing pyramid with the height of the visible length.

(c) The space to be calculated is completely filled by illumination volumes. Defining the triangle as a *caustic triangle* which is the intersected area between an illumination volume and a scan plane (see Fig.2), every point on the scan plane is included in at least one caustic triangle because every edge of the illumination volume is shared by adjacent illumination volumes (there is no gap between the triangles).

(d) The intensity of light in the caustic triangle is inversely propotional to its area. Usually illumination volumes are very thin, so their caustic triangles are very small; the intensity of scattered light from particles in the caustic triangle is assumed as uniform.

(e) The light arriving at the viewpoint is determined only by the distance between the caustic triangle and the viewpoint, since the density of particles in water is uniform. That is, the intensity can be calculated by an analytical function (exponential function) of the distance.

(f) It is difficult to calculate intensity distribution within the water due to skylight as it incidents onto wave surface from various directions. This light after passing through wave surfaces is treated as an ambient light, the intensity of which is defined by the exponential function of its depth.

3 OUTLINE OF THE PROCEDURE

First, we prepare the following: The water surface is subdivided into an $N \times M$ mesh, and illumination volumes, the number of which is $2NM$, are generated by subdivision of the mesh. The refracting direction of incident light at each lattice point is calculated where the direction is influenced by the wave shapes. A 2-dimensional array is prepared to store these refracting directions. A Z-buffer to store depths on the scanline and an accumulation buffer to store intensities are prepared. The visible length is calculated by using the extinction coefficient.

The outline of the procedure is as follows:

(1) Calculate the normal vector of the wave at each lattice point and store the refracting direction at each point.

(2) Execute the following processes for each scanline which moves from top to bottom :

(2.1) After hidden surface removal, store the depths of objects(or water surface) in Z-buffer.

(2.2) Calculate the intersection points between the edges of illumination volumes and the scan plane.

(2.3) Execute the following processes for each caustic triangle. Hence, caustic triangles lying outside of the view pyramid and behind objects tested by the Z-buffer are discarded.

(2.3.1) Calculate the maximum intensity of scattered light for the caustic triangle

(2.3.2) Calculate the projected range of the triangle on the screen by means of perspective transformation, calculate the intensity at each pixel in the range by using linear interpolation, and store them in the accumulation buffer.

(3) Display every pixel color stored in the accumulation buffer.

The method described above is superior to previous methods by virtue of the following: In the previous methods (e.g., Watt [22] employed Nishita's method [16]), ray-tracing is used for the calculation of shafts of light in space. There are huge number of illumination volumes (e.g., more than 100,000 triangular prisms in our examples). This results in the number of intersection tests between rays and illumination volumes becoming huge because intersection tests are required for each pixel on the screen, and the intersection test for illumination volumes is included in the loop of the ray. The proposed method, however, is very effective because the calculation of intensity at every pixel is done in the loop of each illumination volume.

4 INTENSITY CALCULATION

Let's consider the light arriving from point Q on an object or a wave surface; the light is influenced by scattering/absorption of particles along its ray. As the intensity within a caustic triangle is assumed as uniform, intensity at each pixel in the projected range of the triangle can be calculated as follows: The intensity at each pixel is calculated by multiplying the intensity of the triangle and the intersection length between the ray and the triangle, and is accumulated in the accumulation buffer. Let's discuss the detail of the method.

The scattered intensity from particles at point P on the ray is given by (see Fig.1)

$$I_P(\lambda) = (I_i T(\theta_i, \theta_t) F_p \beta(\phi, \lambda) exp(-c(\lambda) l_s) + I_a)\rho, \quad (1)$$

where λ is the wavelength, I_i the intensity of incident light onto the water surface (I_i the irradiance of sunlight just above the water surface), T the transmittance of the incident light which obeys Fresnel's law of reflection, θ_i and θ_t the incident angle and the refracted angle, respectively; these angles obey Snell's law. β a volume scattering function (this is the function of angle ϕ between the incident direction and scattering direction: see Fig.1), $c(\lambda)$ the attenuation coefficient of light which expresses the ratio of lost energy of light when the light travels a unit length, ρ the density, and I_a ambient light (see 2.3(f)). The color of the water is determined by $c(\lambda)$ and β. (Data of β and c used in this paper is employed from [8].) F_p is the flux ratio between the intensity at just beneath the water surface and at point P, $F_p = S_u / S'_s$, which is equivalent to form factor in the radiosity method. S_u and S'_s are areas of cross sections at water surface and at point P, respectively. S'_s is determined by the area of the caustic triangle on the scan plane and cosine of angle δ (see Fig.3). That is,

$$S'_s = S_s cos\delta. \quad (2)$$

As the intensities of scattered light from every particle on the ray are attenuated, the intensity at the viewpoint, I_v, is given by

$$I_v(\lambda) = I_q(\lambda) exp(-c(\lambda) L) + \int_0^L I_p exp(-c(\lambda) l) dl, \quad (3)$$

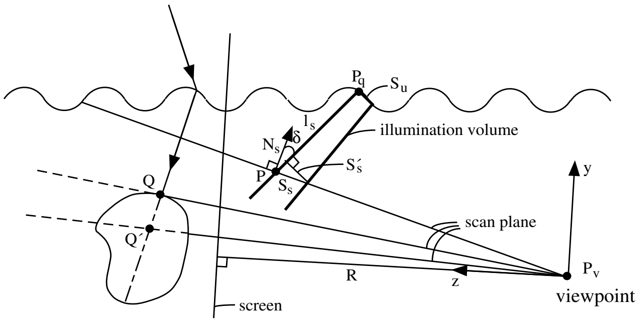

Figure 3: Intensity calculation and shadow test on a scan plane.

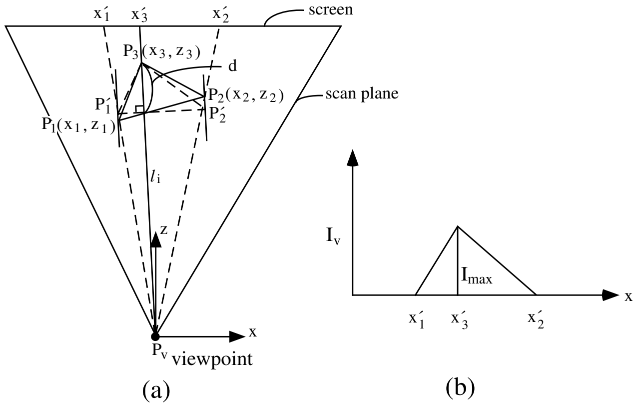

Figure 4: Calculation of intensities due to a caustic triangle.

where I_q is the reflected intensity from the object (or refracted light from wave surface), I_p the scattered light from particles, and l the distance between P and the viewpoint, L the distance between the object and the viewpoint.

In the case of point Q being on the wave surface and θ_t is less than the critical angle(48.6°), I_p is given by

$$I_q(\lambda) = T(\theta_i, \theta_t)(I_{sun}\delta(\theta_i - \theta_t) + I_{sky}(\theta_i)), \quad (4)$$

where I_{sun} is the incident intensity, $\delta(\theta_i - \theta_t)$ the delta function (1 for $\theta_i = \theta_t$). $I_{sky}(\theta_i)$ is the intensity of skylight in angle θ_i (see [15]).

In the case where the incident ray from the viewpoint is larger than the critical angle (see P_2 in Fig.1), the light from the underside of the water surface reflects perfectly, that is, the scattered light from particles on the reflecting line is reflected. As this reflected light is attenuated when it arrives at the viewpoint, it is not necessary to calculate precisely. These intensities depend on each reflecting angle, then these values are precalculated for several angles and are stored in a table-lookup.

When the light does not intersect with any objects (or the water surface) any great distance away, the visible length is set as $L = \infty$. The attenuation effect in water is large, the light from distant position doesn't arrive at the viewpoint; this means the integration range(interval) is limited. For a given tolerance ϵ, the integration range, i.e., visible length L_∞, can be defined by the relation of $exp(-c(\lambda)L) < \epsilon$ ($c(\lambda)$ for blue is used because it is small). I_q at this point is assumed as zero.

The calculation method for reflected light from a point Q on an object is described in section 4.2.

4.1 Intensity calculation using caustic triangles

The calculation for the intensity distribution of light in space is similar to that of shadow calculation using *shadow volumes*. In the method proposed here, illumination volumes are prepared to calculate the intensity distribution of light. These are scan converted (i.e., extracting intersected triangles on the scan plane), and the intensity distribution are calculated scanline by scanline.

As every point on the ray is included in at least one caustic triangle, the 2nd term of equation (3), I_v', is rewritten as follows:

$$I_v'(\lambda) = \sum_{i=0}^{K}(I_{pi}d_i + I_q H_i)exp(-c(\lambda)l_i), \quad (5)$$

where K is the number of caustic triangles on the visible range(interval) of the ray, I_{pi} the average intensity of each caustic triangle (per unit length), d_i the intersected length with each triangle and the ray, l_i the distance between each triangle and the viewpoint (see Fig.4). H_i is binary function (0 or 1); 1 in the case when the triangle i intersect with objects (visible surface or water surface)(see 4.2). Triangles whose depths are larger than the visible length are discarded. Equation (5) gives I_v for one ray. By taking into account every triangle on the scan plane, I_v at every pixel on the scanline can be calculated. The shaft of light in space is obtained by applying this to every scanline.

(1) Extraction of caustic triangles on the scan plane

The coordinates of each caustic triangle on a scan plane can be calculated as follows (see Fig.3): Let's consider a lattice point $P_u(x_u, y_u, z_u)$ on a wave surface and unit vector of its refraction $\mathbf{Lu}(L_x, L_y, L_z)$ in an eye-point coordinates system. If the distance between the viewpoint and the screen is R, y coordinate of the scan line is y_s, and x, y, z components of the normal vector $\mathbf{Ns}$ are $(0, 1, -r_s)$, here $r_s = \frac{y_s}{R}$. The length l_s between lattice point P_u and the intersection point between the scan plane and the refracted ray is given by

$$l_s = \frac{y_u - r_s z_u}{L_y - r_s L_z}, \quad (6)$$

where the refracted ray intersects with the scan plane when the numerator is positive (P_u exists below the scan plane when it is negative). By using the obtained l_s, intersection point P can be calculated by using the equation $\mathbf{P} = l_s\mathbf{Lu} + \mathbf{Pu}$. The denominator is propotional to the cosine of angle δ between the normal of the scan plane and the refracted vector (see Fig.3), and $cos\delta$ is given by

$$cos\delta = (L_y - r_s L_z)a_s, \quad a_s = (1 + r_s^2)^{-1/2}, \quad (7)$$

where a_s is constant for the same scan plane. Eqs. (6) and (7) are used effectively because $cos\delta$ and l_s can be used in equations (2) and (1), respectively.

(2) Calculation of scattered light within a caustic triangle

The intensity of light in a caustic triangle can be calculated by using its area. If we calculate the areas of every caustic triangle on each scan plane, it is time consuming. So we use the approximation method for calculation of the intensity of light at each pixel, in which area calculation can be avoided.

Let's consider the first term in eq. (5). Fig.4 (a) shows the top view of a scan plane in eye-coordinate systems. Let's consider caustic triangle $P_1(x_1, z_1)$, $P_2(x_2, z_2)$, $P_3(x_3, z_3)$ on the scan plane, where the area of the triangle is S_s, and the vertices are sorted in order $x_1 \le x_3 \le x_2$. As shown in Fig.4(a), points P_1 and P_2 are moved parallel with the line P_vP_3, and P_1P_2 is perpendicular to P_vP_3; these moved points are denoted by P_1' and P_2'. In this case, the area of $P_1P_2P_3$ equals that of $P_1'P_2'P_3$ (i.e., S_s). The area of the triangle, which is half of the parallelogram, is expressed by $\overline{P_1'P_2'} \times d/2$. We need the intersected length between the ray and the triangle to calculate intensity. When the ray passes through P_3, the length is maximum. The length of the intersection between the triangle and P_vP_3, d, is expressed by $\frac{2S_s}{\overline{P_1'P_2'}}$. The length $\overline{P_1'P_2'}$ can be obtained by adding distance P_1' from the line P_vP_3 and the distance P_2' from the same line, and is expressed by $((x_2-x_1)-(z_2-z_1)(\frac{x_3}{z_3}))/b$, where $b=\sqrt{1+(\frac{x_3}{z_3})^2}$. Let's denote projected x-coordinates of triangle $P_1P_2P_3$ on the screen as x_1', x_2', x_3'. After projection, the triangle affects to the intensity at pixels between x_1' and x_2', and the intensity at x_3' has a maximum value because the intensity depends on the intersected length of the ray and the triangle. That is, as shown in Fig.4(b), the intensity distribution between x_1' and x_2' is approximated by two liner functions. For simple calculation, x_3' is slightly moved to the center of pixel which includes x_3'. The distribution function can be determined uniquely by the maximum intensity. As described previously in 2.3(a), for calculating the intensity of a shaft of light only direct sunlight is taken in account; the second term in eq.(1) can be neglected. By assuming I_s' except for F_p in the first term of eq.(1) ($I_p = I_s'F_p$), I_p is expressed by $I_s'\frac{S_u}{S_s}$. Then the maximum intensity of $I_{pi}d_i$ of the first term of equation (5) is given by (see 4.2 for 2nd term)

$$I_{max} = I_s'\frac{S_u}{S_s}\frac{2S_s}{\overline{P_1'P_2'}} = \frac{2I_s'S_ub}{(x_2-x_1)-(z_2-z_1)(\frac{x_3}{z_3})}. \qquad (8)$$

That is, S_s is canceled; area calculation of the caustic triangle is unnecessary. As $(\frac{x_3}{z_3})$ is small, b is approximated by $1+(\frac{x_3}{z_3})/2$. For very small $(\frac{x_3}{z_3})$ (i.e., around center on the screen), the intensity of the triangle is inversely propotional to the width (x_2-x_1) of triangle viewed from the viewpoint. Intensity at every pixel in the interval $[x_1', x_2']$ except x_3' is calculated by linear interpolation.

Let's consider a accumulation buffer, $I(x)$ ($x = 1, 2, ...n$: n =screen width), to be stored intensities. After initializing the accumulation buffer, the accumulation process to the accumulation buffer is done by the following quasi-code.

```
d1 = I_max/(x3' - x1');
d = d1([x3'] + 1 - [x1']);
for x = [x1'] + 1, [x3'];
{ if (z(x) > z_max) I(x) = I(x) + d ;
  d = d + d1;
}
 d2 = I_max/(x2' - x3') ;
for x = [x3'] + 1, [x2']
 { if (z(x) > z_max) I(x) = I(x) + d;
  d = d - d2;
 }
```

where [ ] is the symbol of truncated fractions, d the intensity at pixel x. $z(x)$ is the depth at pixel x which is stored in the Z-buffer, and z_{max} the maximum depth of the triangle. In the above process, if the triangle is nearer than the depth of the object at the pixel, the intensity at the pixel is stored in the accumulation buffer like the Z-buffer algorithm.

4.2 Intensity calculation on objects

To calculate intensity of reflected light from point Q on an object, illumination volumes hitting Q have to be extracted; at least one illumination volume exists. In previous methods, the intensity distribution is calculated beforehand and stored in an illumination map. Then they require substantial storage memory, and their shape is limited to polygons: even intensities of invisible parts are calculated. If an illumination map is not used, it is time consuming because intensity tests between objects and illumination volumes are executed at every point on the objects.

However our method makes both illumination map and intersection tests between objects and illumination volumes unnecessary. As shown in Fig.2, if a caustic triangle on a scan plane contains point Q on the object (x and z components are enough to test), the illumination volume including the caustic triangle illuminates point Q. As the depth of the object at each pixel is stored in the Z-buffer, this test is performed simply by comparing the depth of the triangle with the depth of the object.

The intensity of reflected light is determined by the intersection area between the object and the illumination volume (the ratio of focus is determined by the area), the angle between the normal of the object and the ray, and the attribute of its material (such as reflectance). In previous method [22] handling polygonal objects, the intersected area(i.e., a casuistic polygon) between the object and the illumination volume should be calculated. For curved surfaces, such a calculation is difficult. In the proposed method, the density ratio of flux, F in eq.(1), can be calculated by using the area of the caustic triangle on the scan plane instead of the intersected area on the object. The area of the triangle is calculated just for the intensity calculation of objects, in general the area of caustic triangle is not calculated (see 4.1). The intensity of reflected light, I_q, is given by

$$I_q = I_iT(\theta_i,\theta_t)F_p exp(-c(\lambda)l_q)(k_d cos\alpha + k_s(cos\gamma)^n), \quad (9)$$

where k_d and k_s are diffuse reflectance and specular reflectance, respectively. l_q is the distance between the calculation point and the water surface, α the angle between the normal of the object and the incident ray, γ the angle between the viewing direction and the reflection direction, n the specular-reflection exponent: we employed Phong's model.

4.3 Shadow processing

For the case where light sources are located above the water surface, the calculation method for shafts of light within the water are described here. As the shafts of light stream from top to bottom on the screen, the scanlines are processed from the top to the bottom. As the shadow effects of objects located above the water surface are ignored, if the starting point of illumination volume (on the water surface) is visible from the viewpoint, this point is not in shadow; the test is easily done by using values of Z-buffer. When the starting point is outside of the screen, the shadow test (i.e., inclusion test whether objects include the point or not) is done. During the proceeding of scanlines, once the caustic triangle on the scan plane intersects with an object (see Q in Fig.3), the illumination volume is cut away in the next scanline because of it being in shadow. In the case of the triangle existing behind the value of Z-buffer, the illumination volume intersects with some object (see shaded region in Fig.2) or passes through behind the object (see dotted region in Fig.2). In this case, shadow testing is executed. That is, if the illumination volume intersects with some object, the illumination volume is cut away.

In water the extinction coefficient is large in general. Scattered light from particle far from the water surface do not arrive at the viewpoint, so the illumination volume is assumed to have a limited length. The distant part of it, therefore, is cut away as in shadow processing.

5 EXAMPLES

Fig. 5 shows some scenes from an animation of optical effects within water. Fig.(a) shows the shafts of light and caustics on a creature

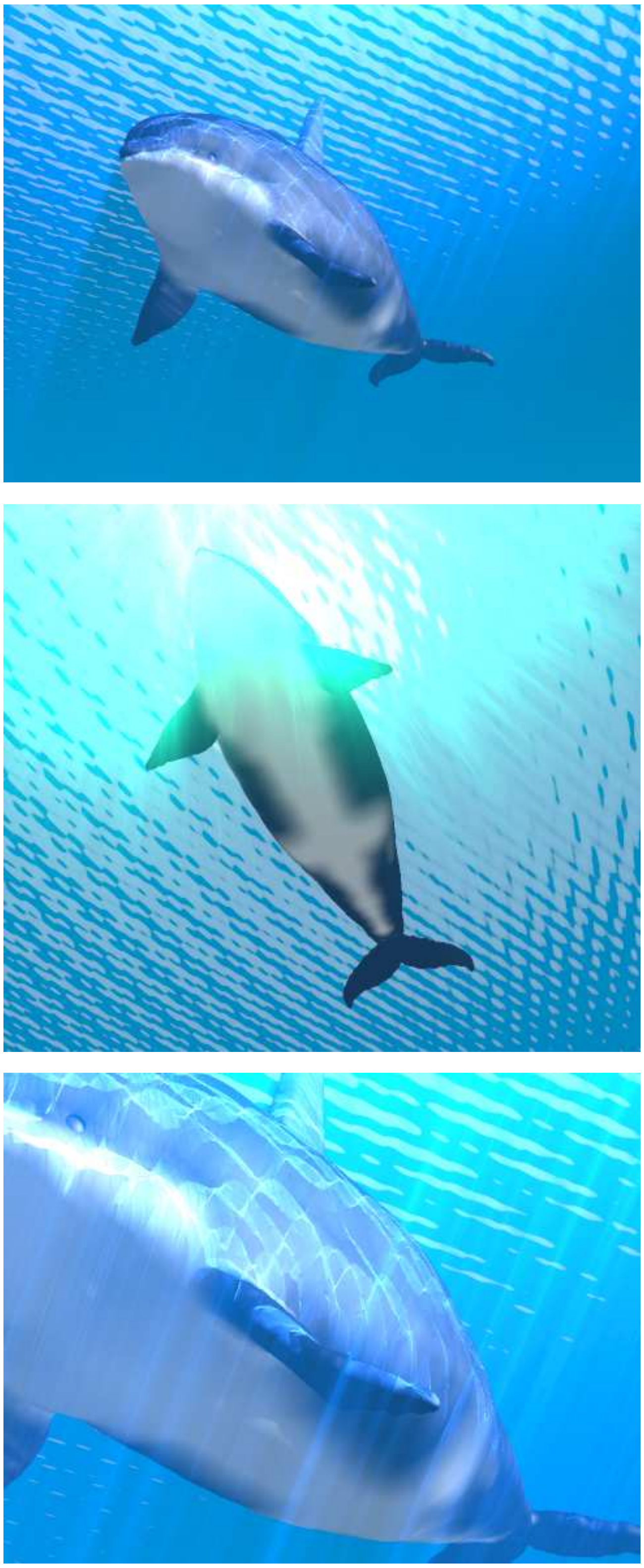

Figure 5: Examples of optical effects within water.

in water; the killer whale consists of 890 metaballs. Fig.(b) shows the killer whale viewed from the bottom of water. Fig.(c) is a close up of Fig.(a) in order to observe caustics on curved surfaces. In these examples, 125,000 illumination volumes are used. The waves are composited by the four different wavelength of waves [9]. The size of the illumination volume is equivalent to $1/36$ of the shortest wavelength. The calculation was done on an IRIS CRIMSON. The computation times for Fig.5 (a) and (b) were 6.2 minutes and 7.4 minutes, respectively (image size=512 $\times$ 400).

6 CONCLUSION

We have proposed an algorithm for rendering the optical effects such as shafts of light, caustics, and the color of the water. As shown in the examples, the proposed method gives us photo-realistic images taking into account scattering characteristic of particles and intensity distribution of light within the water. The advantages of the proposed method are as follows:

(1) For calculation of caustics on objects, the method proposed here doesn't need any illumination map used in previous methods. The method can display caustics on curved surfaces such as metaballs.
(2) The method proposed here can calculate optical effects, shaft of light, caustics, and color of water, effectively by using a scanline Z-buffer and accumulation buffer.
(3) The shadow effects of shafts of light can be calculated simply by using Z-buffer.

Even though we did not show images viewed from above the water surface, the method proposed here can be applied to atmospheres including caustics on objects due to reflected light from water surfaces.

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