Continuous Tone Representation of Three-Dimensional Objects Taking Account of Shadows and Interreflection

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Abstract

The effect of shadows and interreflection created by room obstructions is an important factor in the continuous tone representation of interiors. For indirect illumination, in most cases a uniform ambient light has been considered, even though the interreflection gives very complex effects with the shaded images.

The proposed method for indirect lighting with shadows results in the following advanced points:
1) The indirect illuminance caused by the surfaces of objects such as ceilings, floors, walls, desks, bookcases etc. gives added realism to images.
2) The proposed method is suitable for every type of light source such as point sources, linear sources, and area sources.

CR Categories and Subject Descriptors: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism; I.3.3 [Computer Graphics]: Picture/Image Generation

General Terms: Algorithms

Additional Key Words and Phrases: Shading, interreflection of light, diffuse reflections, area light source, shadows, penumbrae

1. INTRODUCTION

Continuous tone representation of three-dimensional objects is a useful tool for CAD of buildings, machines and various lighting problems. The degree of realism of the shaded image of a three-dimensional scene depends remarkably on the successful simulation of shadowing and shading effects. In order to display three-dimensional objects that look more realistic, researchers have developed techniques for simulating the properties of objects such as reflection, refraction, and transparency (e.g., Ref.[1,2,3]). In most computer-generated pictures, however, ambient light has been treated in a perfunctory way; i.e., a constant rough estimate is used. Particularly when displaying the interior of a room, indirect illumination caused by interreflection is very significant; this component usually forms about 30 percent of the total illuminance and the ambient light is not uniform.

In the traditional lighting design methods for handling interreflection (e.g.,[4,5]), it is assumed that the light sources are point sources, the room is rectangular and empty(i.e., the room is defined by six rectangles and every surface can be seen from every other surface). It is also assumed that the distribution of illuminance is uniform on each surface.

Recently, Jepsen et al. [6] developed a calculation method of interreflection taking into account shadows. This method is, however, only available for the interreflection between two parallel(or perpendicular) surfaces with uniform illumination. Goral et al. [7] also proposed modeling the interaction of light with color bleeding, but only discussed this in an empty room.

The shading method proposed in this paper is applicable to an arbitrary shaped room; it calculates shadows caused by obstructions such as desks, bookcases in the room, and it allows direct and indirect illumination from various types of light sources. Color bleeding is not considered.

Direct illuminance calculation for various types of light sources such as point sources, linear sources, area sources, and polyhedron sources is performed using the method of Ref.[8,9]. Area light sources illuminating a computer room are calculated as an example in order to show the effect of interreflection.

2. PREPARATION AND OUTLINE OF PROCEDURE

We assume the following for this discussion.
1) The algorithm described here applies to objects composed of several convex polyhedra. The normal of a face consisting of a polyhedron is an outward-pointing vector.
2) All surfaces of polyhedra are perfect diffuse

surfaces(i.e., Lambertian reflecting surfaces).

3) The types of light sources handled here are point light sources, linear light sources, area light sources and polyhedron light sources. Except for point sources, the distribution characteristics of sources are Lambertian distribution. The shape of area sources and polyhedron sources are convex.

4) The calculation of direct illuminance and interreflection illuminance is applied to diffuse reflection not to transparency and specular reflection.

The fundamental ideas of the algorithm are as follows:

1) If the calculation of shadows on surfaces is executed by using the point by point method, it takes considerable computation time. In order to save time, the shadow areas(penumbrae and umbrae) are predetected before scanning for hidden surface removal. The shadows are determined in the following manner. First, shadow volumes for penumbrae and umbrae formed by a convex polyhedron and a light source are obtained; these shadow volumes are named penumbra volumes and umbra volumes, respectively[8,9]. Then, the penumbrae (or umbrae) on each face are obtained as the intersection areas of penumbra(or umbra) volumes and the face. We give a brief description of shading calculations for area(or polyhedron) light sources in Appendix.

2) The interreflection of light is obtained by the following method. The faces in a room, such as walls and objects, are subdivided into pieces (elements) called subfaces(or zones). The interreflection of light at the vertices of each subsurface is calculated before scanning for shading and each point on the screen is obtained by interpolation.

The outline of the procedure is as follows:

(1) Input of three-dimensional objects.
(2) Subdivision of faces into subsurfaces.
(3) Classification of faces of each polyhedron for shading. Faces are classified into three classes; faces receiving light from the whole region of the source, faces receiving light from a part of the source, and faces receiving no light from the source.
(4) Obtaining penumbra volumes and umbra volumes.
(5) Calculation of penumbrae and umbrae on each face.
(6) Calculation of interreflection of light.
(7) Determining priority of visibility for a given viewpoint.
(8) Hidden surface removal and calculation of direct illuminance at each point on a screen(see Ref.[10,11] for hidden surface removal and anti-aliasing).

3. CALCULATION OF INTERREFLECTION OF LIGHT

3.1 Traditional Calculation Method of Interreflection of Light

In illumination engineering, calculation methods for interreflection already have been developed for an empty room. In the traditional methods mentioned before, the surfaces of the room are subdivided into some subsurfaces such as rectangles. This subdivision is similar to mesh generation in finite element analysis. The total illumination(direct and indirect) are obtained by solving the matrix equation.

The traditional methods ignoring the shadow effect are as follows: Let's consider the interreflection between the two faces S_i and S_j (see Fig. 1). The equation of interreflection is given by

$$E_i = E_{0i} + \int_{A_j} \rho_j e_0(i,j) E_j \, dA_j \quad , \qquad (1)$$

where E_{0i} = direct(initial) illuminance at P_i,
E_i = final illuminance at P_i,
E_j = final illuminance at P_j,
ρ_j = reflectance at P_j,
A_j = area of S_j,
$e_0(i,j)$=radiative exchange factor
r_{ij} = distance between P_i and P_j
θ_i(or θ_j)= the angle between normal of S_i(or S_j) and line segment P_iP_j

The second term in equation (1) indicates the indirect illuminance component.

It is difficult to solve the above integral equation, therefore, some solving methods such as Fourier Series analysis[6] are used. In this paper, the equation is solved by employing a finite approximation. That is, an enclosure is subdivided into subsurfaces which are rectangular planes.

If a room consists of n subsurfaces, the illuminance is given by

$$E_i = E_{0i} + \sum_{j=1}^{n} \int_{A_j} \rho_j e_0(i,j) E_j \, dA_j \, . \qquad (2)$$

If illuminance and reflectance are uniform over the entire extent of subsurfaces, E_i is expressed by multiplying $1/A_i$,

$$E_i = E_{0i} + \sum_{j=1}^{n} \rho_j F_{ij} E_j \, , \qquad (3)$$

where

$$F_{ij} = 1/(A_i) \int_{A_j} \int_{A_i} \cos\theta_i \cos\theta_j / (\pi r_{ij}^2) \, dA_i \, dA_j$$

(F_{ij} is called Form Factor)

A_i = area of S_i

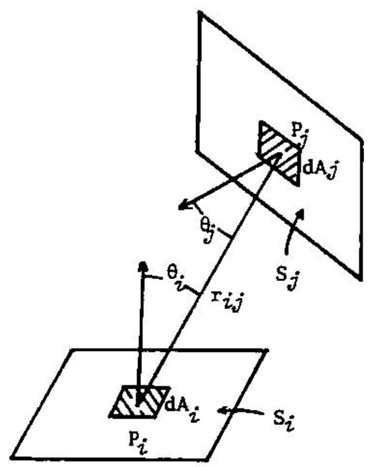


Fig.1 Interreflection between two faces.

and when $\cos\theta_i \leq 0$ or $\cos\theta_j \leq 0$, $F_{ij}=0$. Let $d_{ij}=\rho_j F_{ij}$, illuminance can be obtained by solving the following simultaneous equation of n unknowns.

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdot \\ E_n \end{bmatrix} = \begin{bmatrix} 1-d_{11} & -d_{12} & -d_{13} & \cdot & -d_{1n} \\ -d_{21} & 1-d_{22} & -d_{23} & \cdot & -d_{2n} \\ -d_{31} & -d_{32} & 1-d_{33} & \cdot & -d_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -d_{n1} & -d_{n2} & -d_{n3} & \cdot & 1-d_{nn} \end{bmatrix}^{-1} \begin{bmatrix} E_{01} \\ E_{02} \\ E_{03} \\ \cdot \\ E_{0n} \end{bmatrix} \quad (4)$$

In equation (4) introduced by the traditional method, we assume that the number of subsurfaces is n, the direct illuminances E_{0i} (i=1,2,.., n) are described by the column vector E_0, the illuminances E_i(i = 1,2,...,n) are described by the column vector E and D is coefficients matrix. The illuminance of each subsurface is obtained by

$$E = D^{-1} E_0 \ . \quad (5)$$

The algorithm proposed here accounts not only for the shadows caused by room obstructions such as desks, bookcases but also for the interreflection between these objects. Therefore, we modify the coefficients matrix D in order to calculate shadow effects, and increase the order n of D(D is n × n matrix) because of the following two reasons.

1) Subdividing of not only faces of a room, but also the faces of objects in the room in order to account for reflection from faces of objects in the room.
2) The illuminance distribution is complicated due to shadows.

3.2 Calculation of interreflection taking into account shadows

In order to calculate shadows, a test to determine whether or not objects exist between every pair of subsurfaces is required. Due to the complexity of this test, it is executed only for the four corner points of every subsurface. That is, the shadow test is done between every corner point. Following this method, the illuminance at the corner points of each subsurface is calculated, and the illuminance of each subsurface is assumed to be an average of the illuminance at its four corner points.

Even though n in equation (5) was the number of subsurfaces, n is now assumed to be the number of points. That is, both shadow calculation and illuminance calculation are done at corner points of subsurfaces. We define shadow function v_{ij} for adding shadow influence between corner points P_i and P_j, weighting coefficients w_j for illuminance calculation at P_j, and α_{ij} is given by $v_{ij}w_j d_{ij}$ (i.e., $\alpha_{ij}=v_{ij}w_j\rho_j F_{ij}$; α_{ij} is the element of the coefficient matrix, ρ_j =reflectance, F_{ij}=form factor). The shadow function v_{ij}(v is step function) and the weighting coefficient, w_j, are given by

$$v_{ij}=\begin{cases} 1: \text{ if no objects between } P_i \text{ and } P_j; \\ 0: \text{ if blocked;} \end{cases}$$

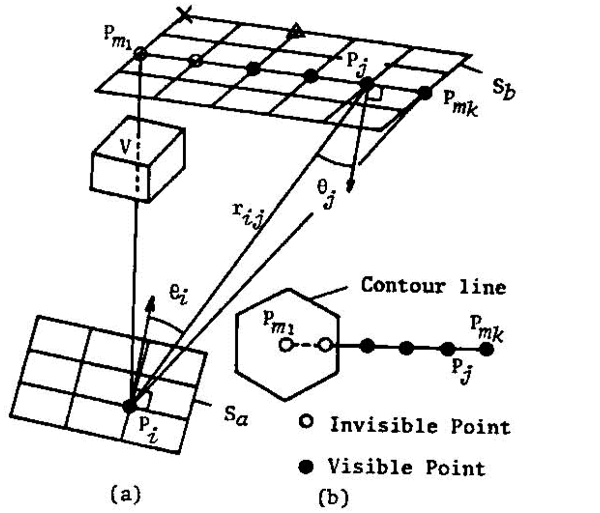


Fig.2 Calculation of shadow function.

$$w_j=\begin{cases} 1/4: P_j \text{ is a corner point of polygon(e.g., "×" in Fig. 2)} \\ 1/2: P_j \text{ is a point on the edge on polygon (e.g., "}\Delta\text{" in Fig. 2)} \\ 1 \ : \text{else.} \end{cases}$$

The subsurfaces are usually very small, then form factor F_{ij} in equation (3) can be approximated as follows:

First, we define the function F(S,P) which means the perpendicular distance between a face S and a point P(X,Y,Z). Let the coefficients of the plane of S be (a,b,c,d) where (a,b,c) are the coefficients of the outward pointing normal to the plane of S.

$$F(S,P) = aX + bY + cZ + d \ . \quad (6)$$

We assume that P_i exists on face S_a, P_j exists on face S_b and the area of subsurface including P_j is A_j. Then the following relations are held: $\cos\theta_i=F(S_a,P_j)/r_{ij}$, $\cos\theta_j=F(S_b,P_i)/r_{ij}$. Therefore, F_{ij} is given by

$$F_{ij}=F(S_a,P_j)F(S_b,P_i)/(\pi r_{ij}^4)A_j \ . \quad (7)$$

Let's explain how to obtain the shadow functions. The calculation of shadow functions v_{ij} are complex. If v_{ij} are calculated by the point by point method, we need much time.

A line segment $P_{ml}P_{mk}$ consisting of P_j(j=ml,...,mk) on a face of a polyhedron, S_b, is handled as a linear light source. As shown in Fig.2, the shadow function for a point P_i on a face of another polyhedron, S_a, is obtained by the following steps:

1) Set V_{ij}(j=ml,....,mk) to 1.
2) Remove polyhedra behind S_a and S_b.
3) Remove the polyhedra not intersecting the triangle $P_iP_{ml}P_{mk}$ by using the bounding boxes of the triangle and the polyhedra.

4) Obtain the polyhedra intersecting the plane of the triangle $P_i P_{ml} P_{mk}$.
5) Extract contour lines(silhouette contours) of the polyhedra viewed from P_i, calculate the intersection between these contour lines and the segment $P_{ml} P_{mk}$.
6) Search the sections(i.e., invisible parts) of $P_{ml} P_{mk}$ enclosed by these contour lines when viewed from P_i, and set $v_{ij}=0$ at every point in these sections(Fig. 2-b shows the contour line of a convex polyhedron and the line segment $P_{ml} P_{mk}$ when viewed from calculation point P_i)

3.3 Calculation of interreflection for a large number of subsurfaces

Equation (5) can be solved by using Gauss Seidel iteration. However, in this method when the number of unknowns, n , is large(e.g., more than 1000), enormous memory capacity(i.e., n × n) is required for the coefficient matrix D. In order to avoid preparing D in the main memory, equation (5) is approximated as follows.

We assume that matrix A consist of α_{ij}(i.e., A=I-D, I=unit matrix) and E_k is the illuminance of the k-th order reflection. E_k is obtained by

$$E_k = AE_{k-1}(k=1,2,\ldots), \qquad (8)$$

where E_0 is direct illuminance. The total illuminance is obtained by summing up every order of the components of illuminance. The values of the higher-order components can be neglected. If we use the equation including the terms up to the K-th order of illuminance, the total illuminance is given by

$$E = \sum_{k=1}^{K} E_k \qquad (9)$$

Once the calculation of direct illuminance E_0, E_1, E_2,.., and E_k are calculated, then the total illuminance is obtained by the summation of these values. By using this calculation method, matrix A is not necessarily stored in the main memory(A is stored in disk) because each of row vectors of A is called from the disk when equation (8) is calculated, Therefore, only three column vectors, E, E_k and E_{k-1}, are required in the main memory.

3.4 Subdivision into subsurfaces

The ceilings, walls, floors and faces of objects within a room are subdivided into subsurfaces. If we take into account all faces of objects in the room for interreflection, the number of subsurfaces become too large, so small faces having low reflectance are ignored.

Let the size of a face to be subdivided be $W_u \times W_v$, and the desired width of a subsurface be W. The width of mesh, w_u and w_v, are obtained by

$$w_u=W_u/[W_u/W+0.5], \quad w_v=W_v/[W_v/W+0.5], \qquad (10)$$

where the symbol [ ] means truncation. Basically the shape of a face to be subdivided is a rectangle or a parallelogram. For other shapes of faces, the minimum rectangle surrounding the face is subdivided and the reflectance at the mesh points outside the original shape of the face are set at zero (points marked "×" in Fig.3-a). Note that the weighting coefficients inner original shape is set as the ratio of the original shapes area in four subsurfaces including the calculation point(e.g., P_1, P_2, P_3 and P_4 in Fig. 3-a are 0.25, 0.5, 1.0 and 0.6, respectively).

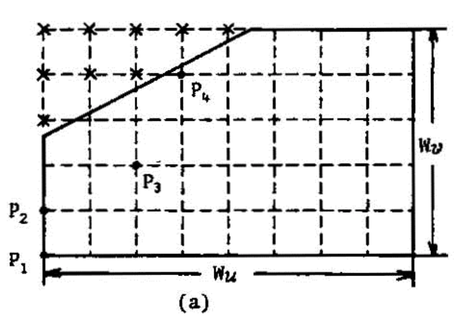
Considering the practical case of the interior of a room, there are doors and windows on the walls. The distributions of their reflectance are different. In order to take into account these effects, different reflectance values are given for the mesh points inside these special areas(e.g., points marked "." in Fig. 3-b). To save memory, only point numbers with changing reflectance values are memorized using a run length encoding method.

3.5 Illuminance at each point on a screen

The illuminance calculation at each point on a scan line is described. Here, we only discuss the indirect component(see Appendix for direct component).

If each subsurface were displayed as a constant intensity, the intensity at the boundaries of the subsurfaces would change stepwise. For smooth shading, indirect illuminance at inner points of the subsurface is obtained by using the linear interpolation in the object space. As shown in Fig. 4-a, let the corner points be $P_{i,j}$, $P_{i+1,j}$, $P_{i+1,j+1}$, $P_{i,j+1}$, and the indirect illuminance at these points $E_{i,j}$, $E_{i+1,j}$, $E_{i+1,j+1}$, $E_{i,j+1}$, respectively. The indirect illuminance at an inner point P is given by

$$E = (1-\alpha)(1-\beta)E_{i,j} + (1-\alpha)\beta E_{i+1,j} + (1-\beta)\alpha E_{i,j+1} + \alpha\beta E_{i+1,j+1}, \qquad (11)$$



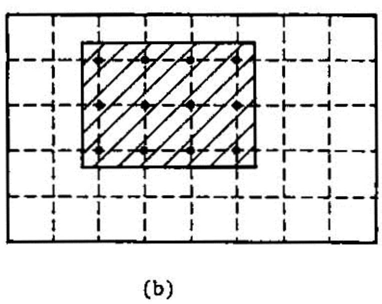
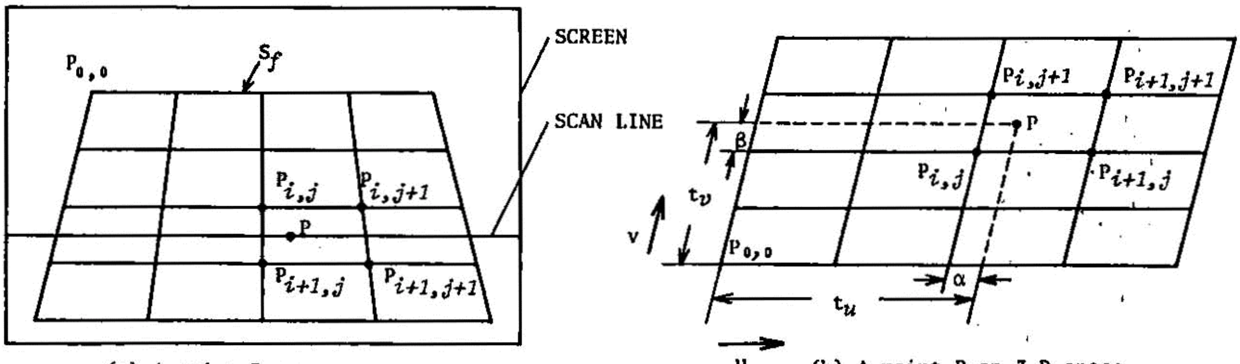


Fig.3 Subdivision of a face into subsurfaces.



(a) A point P on a screen (b) A point P on 3-D space

Fig.4 Illuminance calculation on a screen.

where weighting factors α and β are obtained by the following.

As shown in Fig. 4-b, we assume that u and v are vectors which indicate i-direction and j-direction of mesh of S_f, and their magnitude are w_u^{-1} and w_v^{-1}, respectively.

Subsurface (i,j) containing P is determined by

$$i=[t_u],\ j=[t_v]\ , \qquad (12)$$

where $\quad t_u=(P-P_{0,0})\cdot u\ ,\quad t_v=(P-P_{0,0})\cdot v$

Thus α and β are obtained by

$$\alpha=t_u-[t_u],\ \beta=t_v-[t_v]. \qquad (13)$$

4. EXAMPLES

Fig. 5 shows a computer room illuminated by area light sources; these examples are made in this room. Pictures (a) and (b) are the room illuminated by the light only from the windows; the windows are usually assumed as area sources. (c) and (d) show night scenes illuminated by two ceiling lamps (rectangle sources).

Pictures (a) and (c) are calculated only for direct illuminance, and (b) and (d) are calculated for direct and indirect illuminance; in all these cases, shadow effects are included.

Picture (e) shows the illuminance distributions of the night scene. The upper left side shows the direct and indirect illuminance, the upper right side only shows the direct illuminance; both of them take into account the shadow effect. While the lower left one shows the direct and indirect illuminance but does not consider the shadow effect. The lower right one only shows the indirect illuminance taking account of shadows; in other words, it shows the difference between the upper left and the upper right.

As shown in picture (e), even though there is no direct illumination on the ceiling, the ceiling is bright due to the indirect illumination, and it shows that the indirect component is not uniform due to obstructions within the room. In these examples, the order of reflection, K, is four, because the illuminance distributions for K=4 and K=5 are almost the same, but some differences exist between 3 and 4. The number of subsurfaces, 980, was chosen in the same way.

These examples make clear that accounting for indirect illumination and shadows(especially penumbrae) is an indispensable condition for generating realistic images.

5. CONCLUSION

This paper described a representation method for three-dimensional objects taking account interreflection and shadows.

The following conclusions can be stated from the results.

(1) It requires a terrible amount of computation time to calculate the interreflection at every point, thus faces are subdivided into subsurfaces. The calculation of interreflection is done on subsurfaces before shading at each point on the screen.

(2) Predetermination of the boundaries of penumbrae and umbrae for direct illumination also can be applied to the calculation of indirect illuminance, including shadows, at each subsurface.

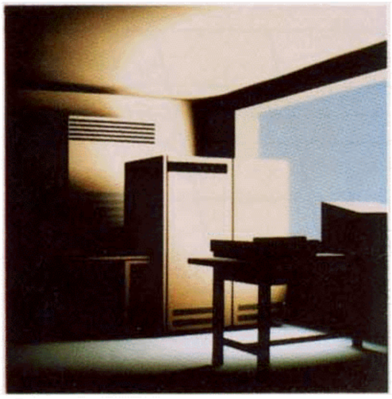
(3) Shading effects between subsurfaces belonging to different faces is calculated at the corner points of subsurfaces. This calculation is done efficiently by handling the series of corner points existing on the same line as a linear source.

(4) When direct illuminance including penumbra and interreflection is calculated precisely, the realism of half-tone representation is much improved.

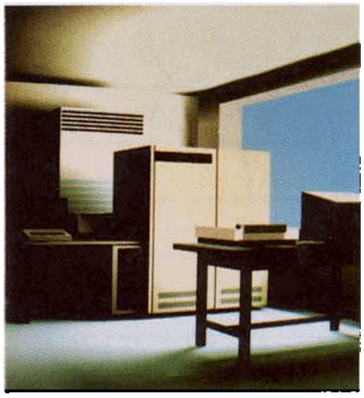
(5) The algorithm can handle direct and indirect illumination for all types of light sources such as point sources, linear sources, area sources, and polyhedron sources. The proposed algorithm thus can be expected to work well for realistic lighting designs.

ACKNOWLEDGEMENTS

We gratefully appreciate the many helpful comments from Laurin Herr of Pacific Interface.



(a)



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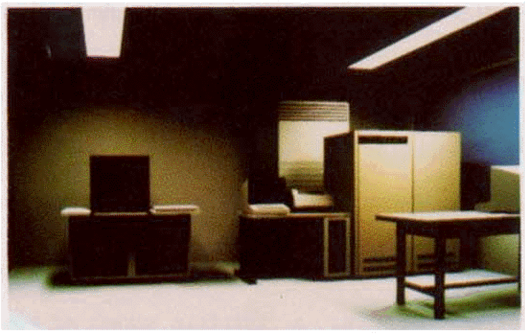
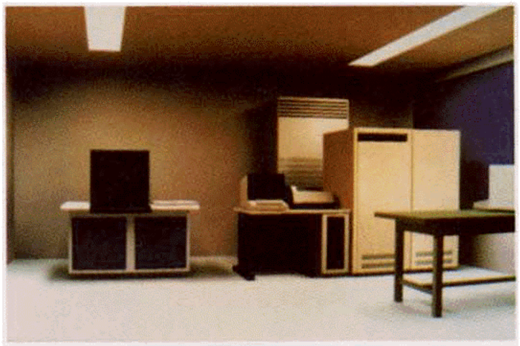
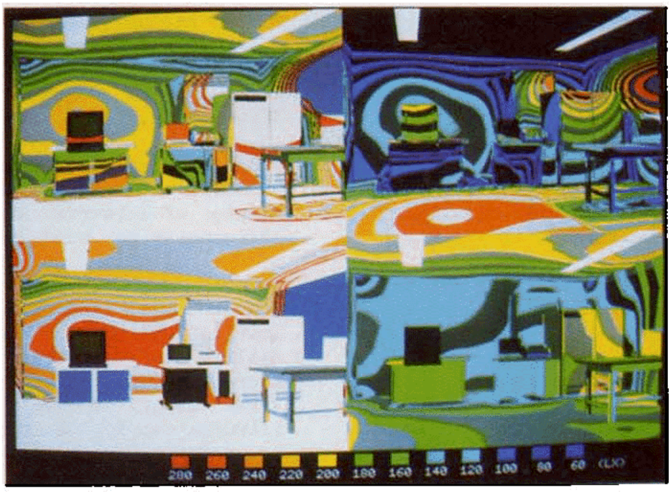


(c)



(d)



(e)

Fig.5 Examples.
(a) and (b) show the room illuminated by the light from a window.
(c) and (d) show night scenes illuminated by two rectangular sources.
(a) and (c) are considered only about direct illumination.
(e) shows the illuminance distribution: upper left; direct+indirect, upper right; direct only, lower left; direct+indirect(no shadows), lower right; indirect only.

Appendix: Shading for finite size light sources

The case of an area light source is discussed here because shading methods for other types of light sources is similar.

(1) Shadow boundaries on faces

In deciding whether or not one convex polyhedron casts its shadow(umbra and penumbra) on another polyhedron, we use the shadow volumes which are formed by polyhedra and light sources; the fundamental idea of shadow volumes was introduced by Crow[12].

The umbra volume and the penumbra volume for an area source are determined by the following method:

We consider a shadow volume U_ℓ which is formed by a convex polyhedron V and one vertex Q_ℓ of a source(as shown in Fig. 6). Then, a penumbra volume is defined as the minimum convex volume surrounding all U_ℓ (ℓ=1,2,..,m), and an umbra volume is defined as the intersection of U_ℓ(ℓ=1,2,..,m).

The same method of application for the area source mentioned above determines umbra and penumbra volumes for linear and polyhedron sources.

The shadow boundary on a face is obtained by the intersection region of the penumbra(or umbra) volume and the face.

(2) Illuminance Calculation

Here we assume that the light sources are composed of uniformly bright surfaces. The illuminance for area light sources can be calculated by the contour integration method for the boundary of the source. As shown in Fig. 7, if the area source has m vertices and an intensity of L, the illuminance at a point P on a face S_f is given by

$$E = L/2 \sum_{\ell=1}^{m} \beta_\ell \cos\delta_\ell \ , \qquad (14)$$

where β_ℓ is the angle between PQ_ℓ and $PQ_{\ell+1}$, δ_ℓ is the angle between the face S_f and the triangle $PQ_\ell Q_{\ell+1}$. The illuminance calculation for polyhedron sources can be obtained by the contour integration method for the contour line of the source when viewed from the calculating point.

For umbrae, the direct illuminance calculation is simplified. Where we consider the shadows from many polyhedra, if the point P is included in at least one umbra, then the direct illuminance at P is zero.

On the other hand, the direct illuminance calculation in the penumbra is more complex. A penumbra area is the region in which the light from the source is partially interrupted by several polyhedra. Therefore, the illuminance at the point P must be calculated by obtaining the visible parts of the light source viewed from P.

The illuminance in the penumbra caused by several polyhedra interrupting light from an area (or polyhedron) source can be calculated by the following method:

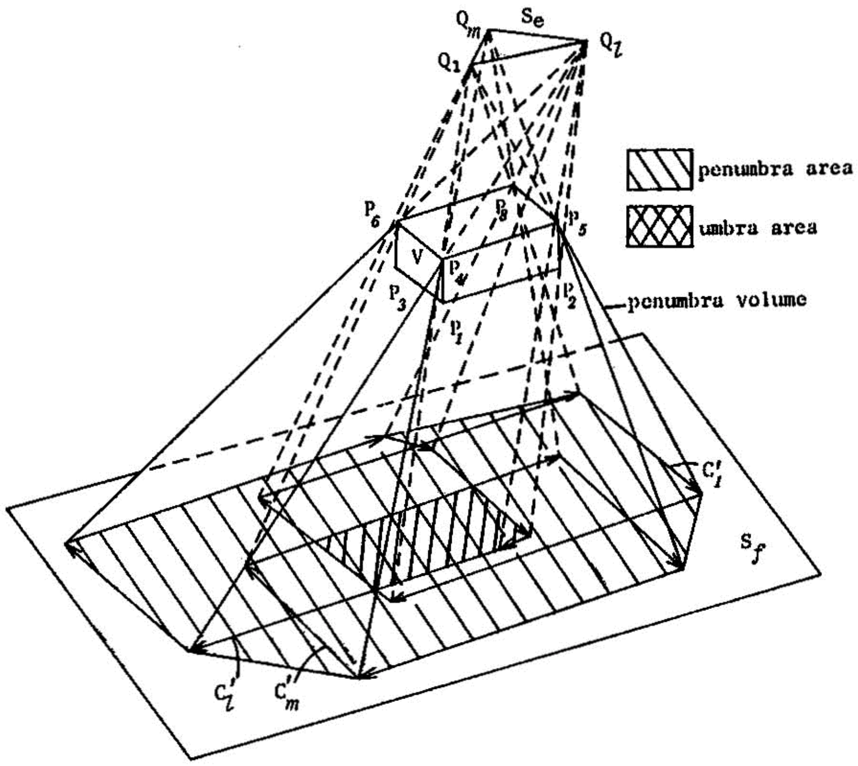


Fig.6 Regions of umbra and penumbra for an area source.

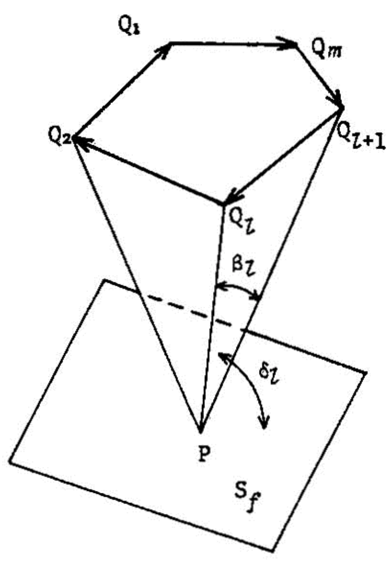


Fig.7 Illuminance calculation for an area source.

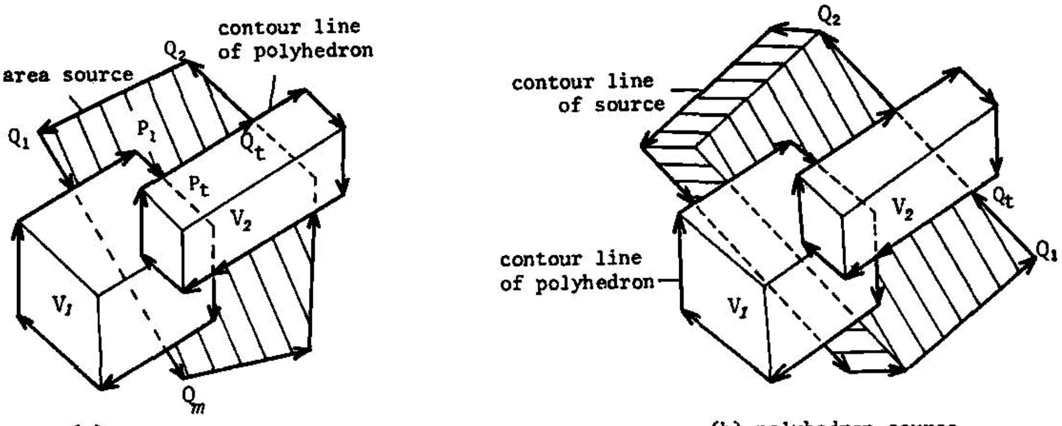


Fig.8 Illuminance calculation in penumbra by using the contour integration method.

The illuminance at P is obtained by summing up the integrated values in the following integrations because it is equal to the integration of the closed region (shaded regions in Fig. 8); integration of the visible segments (e.g., Q_tQ_2 in Fig. 8-a, Q_1Q_t in Fig. 8-b) of the contour line of the source in a counter-clockwise direction, and integration of the segments which exist within the contour line of the source and outside the contour lines of another polyhedra (e.g., P_1P_t in Fig. 8-a), in a clockwise direction.

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