Display Method of the Sky Color Taking into Account Multiple Scattering

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Abstract

Research on the rendering of natural scenes, such as clouds, ocean waves, trees, terrain, grass, and fire, has been growing very popular. This paper reports a shading model for sky color which is indispensable for natural scenes. In most cases, the sky color in outdoor scenes is blue in computer graphics for the sake of simplicity. However, the color of the sky is not simply blue. It changes according to viewing direction, the position of the sun, and conditions of the atmosphere. The sky plays an important role in architectural design and flight simulators. The color of the sky is determined by the scattering and absorption of air molecules, aerosols and other small particles in the atmosphere. In the previous method, only single scattering was taken into account for the sky color calculation. To display realistic images, multiple scattering can not be ignored.

This paper proposes two methods: a fast method to calculate the spectral distribution of sky radiation for single scattering, and an efficient method to calculate the sky color taking into account multiple scattering.

1 Introduction

In computer graphics, natural phenomena such as clouds and waves have often been simulated. Recently, models of atmospheric particles in the atmosphere between light sources (e.g., the sun) and objects have been attracting researchers' attention, the color of the atmosphere being observed as sky color. This paper discusses the method for calculating the sky color. The color of the atmosphere affects the appearance of all objects; for example buildings and mountains in nature are illuminated by direct sun light through the atmosphere and the sky light (i.e., the sky color).

Images generated using computer graphics often have the sky as a background. The color of the sky is not simply blue. The sky color around the zenith is different from the color around the horizon. Moreover, the color changes depending on the time of day (i.e., the sun position). For example, in the daytime the sky looks blue, changing to white near the horizon. However, at sunset or sunrise the sky becomes red near the horizon. This is fascinating. These color variations are caused by the scattering of air molecules and aerosols. The color of the sky is affected by the altitude of the sun, the viewing direction, the height of the observer, conditions of the atmosphere, and the reflected light from the ground. Several methods for calculating the sky color have been proposed. Unfortunately, only single scattering has been considered in those methods. In order to display a more realistic sky, multiple scattering must be considered. It gives us a bluer sky.

The numerical simulation of the atmospheric color considering the scattering of particles can be applied to the following situations. i) the calculation of the sky color in the viewing field: background scenes including buildings and natural scenes (such as mountains, clouds), and flight simulation. ii) Calculation of the sky color in an arbitrary direction. For example, reflection of the sky on the windows of buildings, cars, or water surfaces. iii) Calculation of the luminance distribution of the sky. For example, the appearance of a building illuminated by the sky light, and the incident light to the ground and clouds. iv) Calculation of the color of an object through the atmosphere. For example, the appearance of mountains and buildings far from a viewer (i.e., fog effect).

In this paper, a method for calculating the sky color considering both Rayleigh and Mie scattering is proposed. The method proposed here has the following advantages: (1) fast calculation of single scattering, (2) efficient calculation of the sky color incorporating multiple scattering (considering the light reflected from the ground). The above advantages are realized by using look-up tables which are made using the geometric properties of the atmosphere around the earth.

In section 2, the previous methods are described. In section 3, the basic concept of the proposed method is explained. Then the fast calculation method of single scattering is proposed in section 4, and the calculation method of multiple scattering is proposed in section 5. In section 6, several examples demonstrate the usefulness of the proposed method.

2 PREVIOUS METHODS

The color of the sky can be determined by both scattering and absorption of small particles in the atmosphere. The scattering effects are observed in many natural phenomena such as water color, clouds, fog, smoke, and dust. Several methods have been developed for simulating such phenomena taking into account single scattering. Several methods, other than sky color, have been proposed for simulating the above scattering effects[2] [16] [12] [5].

In this paper, we focus on a method for calculating the sky color viewed from the ground. Several algorithms have been proposed for solving this problem[11] [7] [10]. In Klassen's method, the atmosphere is approximated as multiple layers of plane-parallel atmosphere with uniform density. Kaneda et al. proposed an improved method where the atmosphere is considered a spherical-shell and the density distributions of both the air molecules and aerosols vary exponentially with altitude. Tadamura et al. discussed the relationship between Kaneda's model and the CIE standard[24]. Nishita expanded Kaneda's model by proposing a method for calculating the color of the atmosphere viewed from outer space[17]. Nishita's method simulates the color of the atmosphere around the earth and the effect of blue earth. Basically the method we propose is also an improved method of Kaneda's model, but this method can calculate the single scattering faster by using the fact that the density distribution of the atmosphere is symmetrical with respect to the center of the earth. Moreover, multiple scattering and the light reflected from the ground are also within our proposed method.

Let's discuss multiple scattering. For multiple scattering, no methods have been proposed for calculating the sky color using computer graphics, however, multiple scattering is taken into account for displaying fire and clouds[8] [12] [1] [13] [23]. These methods measure the density distribution that changes arbitrarily in the micro space such as flames, rather than the macro space, such as the atmosphere of the earth. Therefore, the algorithm is complex and requires a lot of memory. These methods are roughly classified into two methods: 1)solution of a large scale matrix equation or 2)Monte-Carlo method. In the field of physics, several methods have been proposed for calculating the sky color resultant from considering multiple scattering[3] [22]. In these methods, multiple layer models or the Monte Carlo method is used to calculate the sky color. The methods, however, calculate the average intensity of the whole sky and therefore the sky color in the viewing direction is not obtained.

The simple and efficient method is used when calculating an atmosphere with a simple regularity of density distribution. The proposed method can calculate multiple scattering efficiently by making a table of cumulative intensity (or transmittance) at sample points aligned with several light directions.

Recently Dobashi et al. proposed a fast display method to display a sky with an arbitrary sun position by recording the intensity distribution of the sky shown with a series of basis functions[4]. Combining both the proposed method and Dobashi's method makes it possible to display the sky quicker.

In order to display the effect of fog, previous methods used a simple exponential function assuming the density of the atmosphere is constant or by using a simple function[14] [6]. The coefficient of the exponential function and the color at infinity should be specified by the user. The color at infinity is equivalent to the color of the atmosphere (i.e., sky color). Using the proposed method, the fog effect caused by the multiple scattering in the atmosphere can be simulated. This strongly affects the viewing ray near the horizon. In this case, the light reflected from the ground is important, and can be calculated by our proposed method.

3 BASIC IDEAS

The color of the sky can be determined by the following factors. (1)The spectral distribution of the sun light, the solar zenith angle, the scattering and absorption properties of particles in the atmosphere, the conditions of the atmosphere (e.g., turbidity), the light reflected from the ground, the scattering and the absorption due to clouds, the solar zenith angle, and view direction. (2) The most important factor is the scattering properties of particles (the phase function), which depends on particle size and the

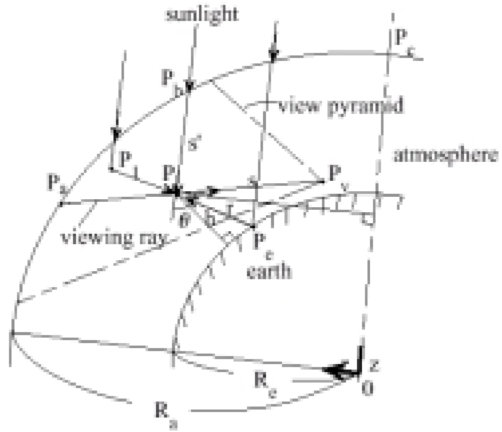

Figure 1: Optical paths for calculation of sky color.

wavelength of light. The atmosphere consists of air molecules, aerosols and the ozone layer. Scattering due to air molecules obeys Rayleigh scattering theory and scattering due to aerosols obeys Mie scattering theory. Aerosols have strong forward scattering. This property affects the bright area of the sky around the sun. (3) The atmosphere is shaped as a spherical-shell and the density distribution of air molecules and aerosols vary exponentially with the altitude. (4) The multiple scattering is not negligible even though the single scattering is more important since the albedo of the atmosphere is relatively low. Especially for light of a short wavelength, the scattered light has a strong affect. Furthermore, the reflected light from the ground is also assumed as one of the factors of the multiple scattering.

For visible wavelengths, absorption in the ozone layer is negligible compared to absorption by air molecules and aerosols. In this paper, the scattering due to clouds is ignored. As shown in Fig. 1, the scattered light at a point, P, on the viewing ray is attenuated through the atmosphere and arrives at the viewpoint P_v. Main optical paths to P are as follows. (1) The sunlight arrives at point P through the atmosphere (path P_bP). (2) The multiple-scattered light of the sunlight arrives at point P (path P_1P). (3) The sunlight after traveling through the atmosphere and being reflected off the ground arrives at P (path P_eP).

Basically, the intensity of the light at the viewpoint can be obtained by integrating the intensity of the scattered light due to particles on the viewing ray. To calculate single scattering, it is necessary to know the attenuated intensity of light arriving at P. This attenuation is determined by the optical length between the top of the atmosphere (P_b in Fig.1) and point P. This requires a great deal of time since the optical length has to be calculated at every sample point on the viewing ray. For the atmosphere of the earth, it is proven that the optical length from the sun to a point in the atmosphere is axisymmetric with respect to the axis parallel to the sunlight (OP_c in Fig.1). This makes it possible to reduce the computation time by precalculating the optical length. To save the calculation time, optical depth is precalculated and stored in a table look-up. We refer this table as "*summed shadow table*".

When calculating the sky light incident to the ground, the sky can be considered a hemispherical dome of large radius with its center being the calculation point[15]. The sky can also be regarded as a spherical dome when the calculation point is in space. In the following example, we call the spherical dome a "*sky sphere*". For multiple scattering, assuming the center of the *sky sphere* at the calculation point on the viewing ray (see Fig.2), the light scattered from the *sky sphere* toward the viewpoint can be calculated. The *sky sphere* is divided into and approximated by several directions. A finite number of directions is sufficient since the distribution of the directional intensity of the sky viewed from a certain calculation point in space does not vary drastically. The total intensity from it, however, changes depending on the altitude of the calculation point (or viewer). We call these directions "*sampling ray directions*". A look-up table for recording the directional intensities of light in the three-dimensional space is prepared. We refer to this table as a "*summed transmittance table*". The atmosphere is subdivided into voxels: assuming voxels so that the direction of one of the edges of a voxel coincides with the sampling ray direction, the cumulative intensity (or transmittance) in its direction is stored at each vertex of the voxel. That is, several sets of the voxels for the sampling directions are prepared. With this method, the attenuation at a certain vertex can be easily calculated incrementally using the result of the previous vertex since the calculation points are aligned in the straight line in the sampling direction. This makes an efficient calculation possible.

4 Fast Calculation Method for Single Scattering

In this section only single scattering is discussed.

4.1 Shading Model for Atmospheric Scattering

The intensity of light arriving at the viewpoint is the attenuated light that is scattered at the point, P, on the viewing ray (see Fig.1). The light arriving at the point, P, is also attenuated due to particles in the atmosphere. Taking into account both Rayleigh scattering and Mie scattering, the intensity at the viewpoint is calculated by the following equation.

$$I_v(\lambda) = \int_{P_v}^{P_a} I_s(\lambda) R(\lambda, s, \theta) \exp(-t(s, \lambda) - t(s', \lambda)) ds \tag{1}$$

where

$$R(\lambda, s, \theta) = K_r(\lambda)\rho_r(s)F_r(\theta) + K_m(\lambda)\rho_m(s)F_m(\theta),$$

λ is the wavelength, P, I_s is the spectral distribution of extraterrestrial solar radiation, s is a variable for the integral (the distance between the point P and the viewpoint P_v), s' is the distance between the point P and the top of the atmosphere, and K_r and K_m are coefficients for the scattering. K_r is inversely proportional to λ^4 and K_m is proportional to $\lambda^{-0.84}$: subscripts "r" and "m" stand for air molecules and aerosols respectively, and components of aerosols are omitted unless necessary in the following. F_r is the phase function (the function of θ shown in Fig.1) and is expressed as $F_r(\theta) = \frac{3}{4}(1 + \cos^2\theta)$. $F_m(\theta)$ has strong forward scattering effects and is the same function used in [18]. $t(s, \lambda)$ is the optical length between point P and the viewpoint, and $t(s', \lambda)$ is the optical length between the top of the atmosphere and point P. t is expressed by the following equation.

$$\begin{aligned} t(s, \lambda) &= \int_0^s (\beta_r(\lambda)\rho_r(l) + \beta_m(\lambda)\rho_m(l)) dl \\ &= \beta_r(\lambda) \int_0^s \rho_r(l) dl + \beta_m(\lambda) \int_0^s \rho_m(l) dl \end{aligned} \tag{2}$$

where β_r is the scattering coefficient (extinction coefficient). β_r is inversely proportional to λ^4. ρ_r is the density at point P and is expressed as $\rho_r =$

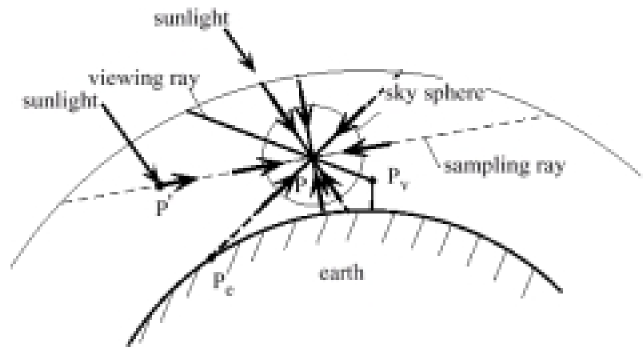

Figure 2: Optical paths for the 2nd order of scattering.

$\rho_{r_0} \exp(-h/H_r)$, where h is the height from the ground, ρ_{r_0} is the density at sea level, and H_r is the scale height.

Since equation (1) can not be solved analytically, the equation is discretized and is calculated by the numerical integration method. The optical length must be calculated at every sample point (i.e., P in Fig.1) on the viewing ray. In order to calculate the sky color precisely, a large number of sample points is required, which increases the computation time. We address this problem by using the geometric properties of the earth and the density distribution. An efficient method using the table generated in the preprocess is described in the next section.

4.2 Optical Length

Let's consider cylindrical coordinate systems, whose axis passes through the center of the earth and is parallel to the sun light. As shown in Fig.1, let the radius be r, and the height from the center of the earth be z. The density distribution of particles in the atmosphere is symmetric to the center of the earth. As the sunlight is parallel light, the optical depth from the top of the atmosphere is axissymmetric with respect to the axis of the cylindrical coordinates (OP_c in Fig.1). That is, points with the same coordinates (r, z), i.e., the ring around the axis, have the same optical depth. The optical depth from the sun can be expressed by the coordinate, (r, z). As shown in equation (2), the optical depth is expressed by two terms; air molecules and aerosols.

Hence $t(s,\lambda) = \beta_r(\lambda)T_r(s) + \beta_m(\lambda)T_m(s)$; the integral part of density for air molecules and aerosols are expressed by functions T_r and T_m, respectively. Both T_r and T_m are stored in the look-up tables. We refer to these tables as *summed shadow tables* because they are used for the attenuation (or shadow) effect of the sun light even though the accumulated densities are recorded. If we record the optical depths in the table, we need the tables for each wavelength because the optical depth t is the function of λ and s. So they are divided into two components. $T_r(s)$ is a two-dimensional array with suffix i and j which correspond to radius r and height z; parameter s is a function of r and z.

Let's consider T_r which denotes the integrated density for air molecules on radius r_i. $T_r(r_i, z_i)$ means the integrated intensity from the top of the atmosphere to point $P_{ij}(r_i, z_j)$, which can be obtained by the following cumulative equation:

$$T_r(r_i, z_j) = T_r(r_i, z_{j-1}) + \int_{z_{j-1}}^{z_j} \rho_r(z)dz \tag{3}$$

where the initial value of the above equation is 0. The existing space of the atmosphere is limited. If we denote the radius of the earth as R_e and the radius of the atmosphere as R_a, T is defined in the following range:

$$R_e^2 \leq r^2 + z^2 \leq R_a^2 \tag{4}$$

where if z is negative, T should satisfy $r > R_e$ because of the shadow region of the earth. Note that T is independent of the viewpoint. So table T is calculated once, and can be used as a data base. It is enough to load their values within the field of view. The field of view is the pyramid clipped by the atmosphere (see Fig.1).

In the integration along the viewing ray, the attenuation of sunlight at a sampling point is calculated by bilinear interpolation of table T. The integration is performed from the viewpoint, and the following optimization is obtained.

To get enough precision, sampling with high frequency is required in a high density area. The sampling intervals are adaptively defined by the function of the height. Use of the *summed shadow tables* and adaptive sampling is 6 times faster than the previous method[9].

For volume rendering of clouds or smoke, the two path methods is used: In the first step, the attenuation of the light source is calculated at each

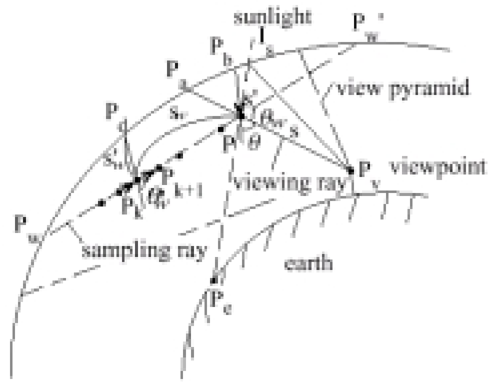

Figure 3: Sampling ray for the 2nd order of scattering.

voxcel, in the second step, the intensity on the viewing ray is resampled[8] [5]. Their methods need three-dimensional arrays to show the attenuation at each voxel, however our method needs only a two-dimensional array for the shadow effect. Furthermore, our method is very effective by using the cumulative processing. This is because we can use the property of density distribution of the atmosphere.

5 CALCULATION OF MULTIPLE SCATTERING

For multiple scattering, the two-pass method[8] [13] [20] is also used like the calculation for single scattering mentioned above. The space is subdivided into a number of volume elements. The first pass deposits flux from the light source and the light scattered at each voxel. The second pass gathers the scattered light along each viewing ray. Max's approach[13] is to allocate the radiosity leaving each volume element into a collection of M direction bins of constant intensity. If we denote the number of voxels as N, and the number of the discrete directions as M, then we have to solve MN matrix equations. Our method is similar to his method, except the direction of voxels (i.e, direction of slice) is different, and our method does not need solution of a matrix equation.

Let's discuss the second order of scattering in this section. For the second order of scattering, the sunlight is scattered at P' incident to point P on the viewing ray, then the light scattered at P reaches viewpoint P_v (see Fig.2). The light from various directions incident to point P. That is, the intensities from every direction w (called the *sampling ray*) should be gathered to P. The intensity reaching the viewpoint P_v, I_v, is given by the following equation (see Fig.3).

$$
\begin{aligned}
I_v(\lambda) &= I_s(\lambda)\int_{P_v}^{P_a}\Big\{R(\lambda,s,\theta)\exp(-t(s',\lambda) \\
&+ \frac{1}{4\pi}\int_{4\pi}dwF(\theta_w)(\int_{P}^{P_w}R(\lambda,s_w,\theta_{w'})\exp(-t(s_w,\lambda) \\
&- t(s'_w,\lambda))ds_w)\exp(-t(s,\lambda))\Big\}ds \qquad (5)
\end{aligned}
$$

where the first term means single scattering, and the second term is the second order of scattering. P_w is the intersection point between the top of the atmosphere and the sampling ray, $\theta_{w'}$ the phase angle between the sampling ray and the sunlight, θ_w the phase angle between the sampling ray and the viewing ray, s_w the distance between the sampling point on the sampling ray and P, $t(s_w,\lambda)$ the optical depth from the sampling point on the sampling ray to point P.

As shown in the above equation, quadruple integration is required because the integration for the optical depths is also necessary. As described in section 3, the incident light to point P is assumed as the light from the *sky sphere* whose center is P (see Fig.2). And the sky sphere is subdivided into several directions called the *sampling ray directions*.

For simplicity, scattering due to air molecules is discussed here. By discretizing the solid angle incident at P to M directions, equation (5) is expressed by the following equation:

$$
I_v(\lambda) = K_r I_s(\lambda)\int_{P_v}^{P_a}(F_r(\theta)\exp(-t(s',\lambda) + \sum_{l=1}^{M}\bar{F}_l g_l(s,\lambda))\rho_r(s)\exp(-t(s,\lambda))ds \qquad (6)
$$

where $g_l(s,\lambda)$ is the function of total transmittance of light incident to point P in sampling direction D_l, which is the total transmittance of the second order of scattering from every point on the sampling ray (PP_w), and is given by the following equation:

$$
g_l(s,\lambda) = \int_{P_w}^{P}R(\lambda,s_w,\theta_t)\exp(-t(s_w,\lambda) - t(s'_w,\lambda))ds_w \qquad (7)
$$

$$\bar{F}_l = \frac{1}{4\pi}\int_{\Delta w} F(\theta_w)dw$$

As described in section 3, the *sky sphere* is subdivided into finite directions. By multiplying the intensity of sunlight I_s to equation (7) gives us the total intensity of the second order of scattering from every point on the sampling ray direction D_l. The above equation $g_l(s,\lambda)$ should be calculated at every sample point on the viewing ray. To calculate g at every sample point during the integration is a waste of time. To save the calculation time, g is precalculated and is stored in a table: the values of g in the viewing pyramid are stored in this paper. These values are stored at lattice points of voxels whose edges are parallel to the sampling ray directions.

Let's denote g at point P_k in direction D_l be $G_k^{(l)}$. $G_k^{(l)}$ is the total transmittance due to particles from P_w to P_k (see Fig.3). After calculating the scattered intensity due to particles on path P_kP_{k+1} and the light attenuated due to the optical depth between it, $G_k^{(l)}$ at point P_k can be calculated by adding both of them to $G_{k-1}^{(l)}$; which is given by the following cumulative equation:

$$G_k^{(l)}(\lambda) = \Delta G_k^{(l)}(\lambda) + G_{k-1}^{(l)}(\lambda)\exp(-\tau_k(\lambda)) \quad (8)$$

where

$$\Delta G_k^{(l)}(\lambda) = \int_{P_{k-1}}^{P_k} R(\lambda,s,\theta)\exp(-t(s,\lambda)-t(s',\lambda))ds,$$

τ_k is the optical depth of the path $P_{k-1}P_k$ and is obtained when $\Delta G_k^{(l)}$ is calculated. s is the distance from a sampling point to P_{k-1}, and s' is the distance from the top of the atmosphere to the sampling point. The *summed shadow table* is used for the calculation of $t(s',\lambda)$ in ΔG_k. By using the above method, the calculation is done efficiently. Note that the values in the opposite direction are also stored because the values from the forward direction (P_w to P_k) and the backward direction ($P_{w'}$ to P_k) are different. The value of g at the arbitrary point P in direction D_l can be calculated by tri-linear interpolation. For the sampling ray directions, eight directions such as the sunlight direction, the perpendicular direction to the sunlight, the horizontal direction to the earth surface, and the zenith direction are employed in this paper. Rayleigh scattering has the following characteristics: the strongest scattering direction is toward the sunlight, while the weakest direction is perpendicular to the sunlight. Hence, these first two directions are selected in our system.

The light reflected from the ground can be calculated as follows. The reflected light has two components; direct sunlight and sky light. Even though the albedo of the ground depends on materials such as soil, trees, and sand, we used average albedos (spectrum reflectivity in this case) used in reference[22], which are weighted averages of each material. For example, the weighted reflectance is obtained from 80% of soil and 20% of green (0.25 in average over visible waves). The ground is assumed to be represented by Lambert's surface, and the reflected light is calculated by use of Lambert's law. See reference[17] for the calculation method of reflected light from sky light. As the initial values of equation (8), the intensities of light reflected from the ground are used only for the sampling rays which intersect the earth (e.g., P_e in Fig.3). The intensity from the ground has a greater effect on the horizontal viewing rays, and is stronger when the sun altitude is high.

Assuming that the number of sampling point is N_1 on the viewing ray, the number of sampling points N_2 on the sampling ray, and the number of sampling ray M, the intensity reaching the viewpoint is the total sum of the scattered light of $N_1 \times N_2 \times M$ points (or voxels). In our experiments, the intensity for one viewing ray includes the intensities at 12,000 points ($M \times N_2 \times N_1 = 8 \times 50 \times 30$). If we take into account the number of sampling points of integral equations (3) and (7), we have to multiply by approximately 100 - the number of sampling points for each numerical integration. Thus the intensity at one pixel (i.e., one viewing ray) is a result of the huge number of sampling points.

Calculation of multiple scattering such as 3rd and 4th order scattering can be done as follows: By assuming point P_k to be point P on the viewing ray, the intensity of the second order of scattering can be obtained. Hence, the intensities of 3rd order scattering for the real viewing ray are obtained. By repeating this process, the higher order scattering is obtained. In general, the intensity levels due to higher order of scattering decrease drastically, so we need calculate only second order scattering.

The intensity due to Rayleigh scattering is inversely proportional to the 4th power of the wavelength. For example, a short wavelight such as blue is more strongly reflected.

6 Examples

Fig.4 shows simple examples of sky color in order to depict the effect of multiple scattering. In Fig.(a) multiple scattering is ignored, but is taken into account in Fig.(b). As this image shows, the sky becomes bluer because of multiple scattering.

Fig.5 (a) and (b) show the terrain model viewed from a point near the ground. Figs. (a) and (c) are in the daytime, (b) and (d) are in the sunset; the solar altitudes are $50°$ and $2°$, respectively. In these images, multiple scattering, reflection from the ground, and the fog effect are considered In Fig.(b), we can see the bright part around the sun due to the strong forward scattering of aerosols. Figs. (c) and (d) show examples of the flight simulator: the view from an aircraft at a high altitude (4,000m). As you can see, the sky color and fog effect in Figs. (c) and (d) are different from Figs. (a) and (b) because the distances and altitudes are different.

The calculation is done on IRIS Indigo2(R4400); the computation time of the sky color in Fig.5 is 10.0 sec for single scattering and 80.0 sec for multiple scattering.

Fig.6 shows the reflected sky color from buildings having reflective surfaces. Figs. (a) and (b) are in the daytime and in the evening, respectively. The solar altitudes are $50°$ and $2°$, respectively.

7 CONCLUSION

We have proposed an algorithm for a physical based image synthesis of sky color. As shown in the examples, the proposed method gives us photo-realistic images which incorporate multiple scattering. The advantages of the proposed method are as follows:

(1) The spectrum of the sky is calculated by taking account of scattering/absorption due to particles in the atmosphere. Calculation for single scattering is accelerated by taking advantage of the fact that the density distribution of the atmosphere is symmetric with respect to the center of the earth. To save calculation time, optical depth is precalculated and stored in a table.

(2) The proposed method can calculate multiple scattering efficiently by making a table of cumulative intensity (or transmittance) at voxels

aligned with several light directions (*sampling ray directions*). The attenuation at a certain vertex can be easily calculated incrementally using the result of the previous vertex, because the calculation points are lined straight in the sampling direction. This reduces the computation time drastically. The reflected light from the ground are also accounted for by handling it as multiple scattering.

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Figure 4: Effect of multiple scattering.

Figure 5: Examples of sky color and fog effect.

Figure 6: Examples of the reflected sky color.