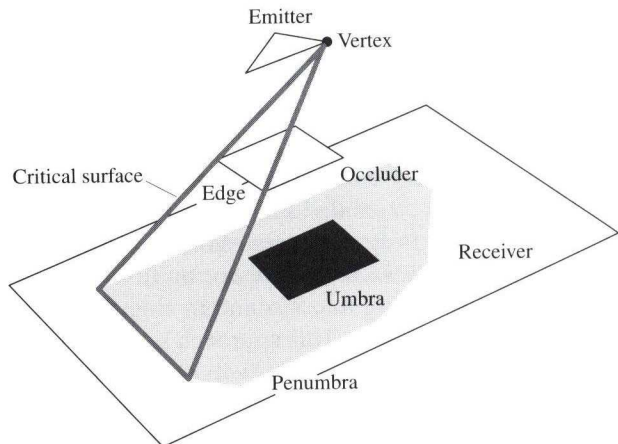


To predict the position of the discontinuities shadow detection algorithms are used and the problem is usually couched in terms of visual events and critical surfaces. Two types of visual events can be considered VE and EEE. VE or vertex–edge events occur when a vertex of a source ‘crosses’ an edge of an occluding polygon known in this context as a receiver. Figure 11.19 shows the interaction between a vertex of a triangular source and an edge of a rectangular occluder. The edge and vertex together form a critical surface whose intersection with a receiving surface forms part of the outer penumbra boundary. For each edge of the occluder a critical surface can be defined with respect to each vertex in the source. We can also define EV events which occur due to the interaction of a source edge with a receiver polygon.

VE events can cause both  $D^1$  and  $D^2$  discontinuities as Figures 11.20 and 11.21 demonstrate. Figure 11.20 shows the case of a  $D^1$  discontinuity. Here there is the coincidence that the edge of the occluder and the source are parallel. Both vertices  $V_1$  and  $V_2$  contribute to the penumbra. As we travel outwards from the umbra along path  $xy$ , the visible area of the source increases linearly and the radiance exhibits piecewise linearity or  $D^1$  discontinuities. A  $D^2$  discontinuity caused by a VE event is shown in Figure 11.21. In this case, a single vertex of the light source is involved along the path  $xy$ . As we travel outwards from the umbra the visible area of the source increases quadratically and the radiance exhibits  $D^2$  discontinuities.

EEE or edge–edge–edge events occur when we have multiple occluders. The important difference here is that the boundary of the penumbra – the critical curve – is no longer a straight line as it was in the previous VE examples but a conic. The corresponding discontinuities in the radiance function along the curve are  $D^2$ . Also the critical surface is no longer a segment of a plane but is a (ruled) quadric surface.

Visual events can occur for any edges and vertices of any object in the scene. For a scene with  $n$  objects there can be  $O(n^2)$  VE critical surfaces and  $O(n^3)$  EEE critical surfaces. Because of the cost and the higher complexity of EEE event approaches have concentrated on detecting VE events.



**Figure 11.19**

VE event: the edge of the occluder and a vertex of the emitter form a critical surface whose intersection with the receiver forms the outer boundary of the penumbra (after Nishita and Nakamae (1985)).

Figure 11.20  
 Event causing a  $D^1$   
 discontinuity (after Lischinski  
*et al.* (1992)).

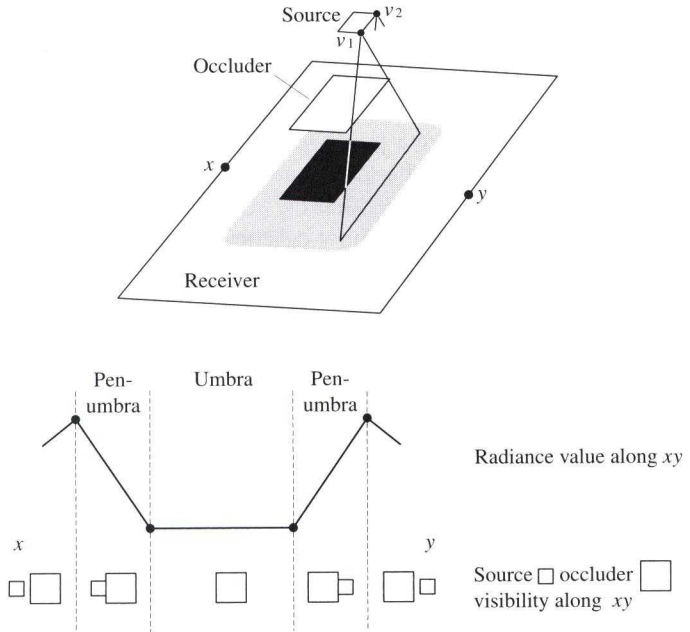
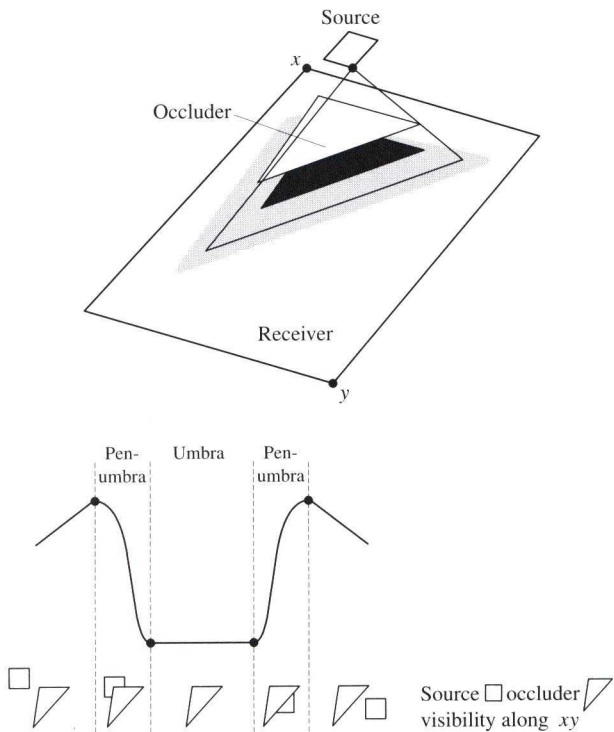


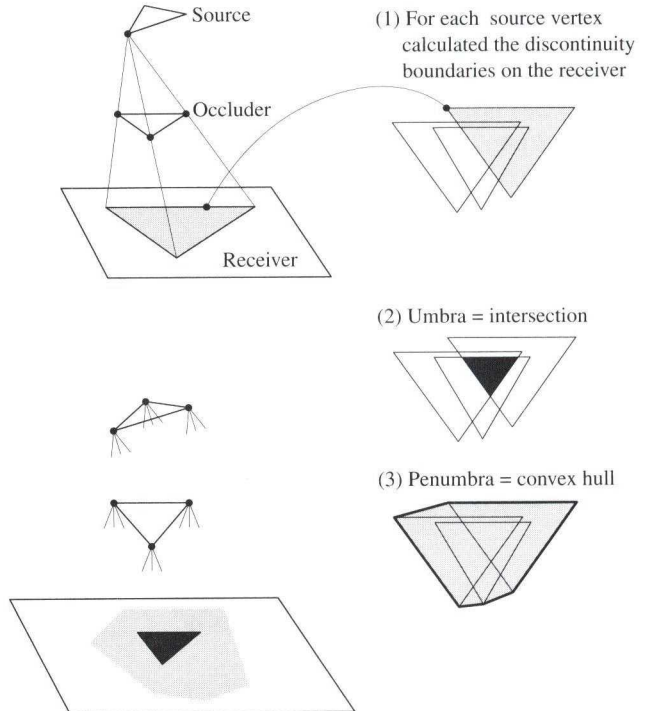
Figure 11.21  
 Event causing a  $D^2$   
 discontinuity (after Lischinski  
*et al.* (1992)).



A straightforward approach by Nishita and Nakamae (1985) explicitly determines penumbra and umbra boundaries by using a shadow volume approach. For each object a shadow volume is constructed from each vertex in the light source. Thus, there is a volume associated with each light source vertex just as if it were a point source. The intersection of all volumes on the receiving surface forms the umbra and the penumbra boundary is given by the convex hull containing the shadow volumes. An example is shown in Figure 11.22.

We will now describe in some detail a later and more elaborate approach by Lischinski *et al.* (1992) to discontinuity meshing. This integrates discontinuity meshing into a modified progressive refinement structure and deals only with V (and EV) events. This particular algorithm is representative in that it deals with most of the factors that must be addressed in a practical discontinuity meshing approach including handling multiple light sources and reconstruction problems.

Lischinski *et al.* build a separate discontinuity mesh for each source, accumulating the results into a final solution. The scene polygons are stored as a BSP tree which means that they can be fetched in front-to-back order from a source vertex. For a source the discontinuities that are due to single VE events are located as follows. Figure 11.23 shows a single VE event generating a wedge defined by the vertex and projectors through the end points of the edge,  $E$ . The event is processed by fetching the polygons in the order  $A$ ,  $B$  and  $C$ .  $A$  is nearer to the source than  $E$  and is thus not affected by the event. If a surface ( $B$  and  $C$ ) faces the source then the intersection of the wedge with the surface adds



**Figure 11.22**  
Umbra and penumbra from shadow volumes formed by VE events.