# **Near-Invariant Blur for Depth and 2D Motion** via Time-Varying Light Field Analysis Yosuke Bando<sup>1,2</sup> Henry Holtzman<sup>2</sup> Ramesh Raskar<sup>2</sup> <sup>1</sup>Toshiba Corporation <sup>2</sup>MIT Media Lab

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## Overview



## Contributions

- Proof of near-optimality of focus sweep in depth and 2D motion invariance and in high-frequency preservation
- Comparison of all the existing computational cameras for deblurring via 5D (4D light field + 1D time) analysis
- Prototype camera demonstration

#### Related Work Prototype

Focus sweep speed

		(a)	Defocus deblurring.	w	Focus sweep speed
Camera design	4D kernel $k_a(\mathbf{x}, \mathbf{u})$		Squared MTF	High freq. preserv.	MTF invariance
	Integration surface	Integration window	$ \hat{\phi}_{s}(\mathbf{f_{x}}) ^{2} =  \hat{k}_{a}(\mathbf{f_{x}}, -s\mathbf{f_{x}}) ^{2}$	$\min_s  \hat{\phi}_s(\mathbf{f_x}) ^2$	$\min_s  \hat{\phi}_s ^2 / \max_s  \hat{\phi}_s ^2$
Upper bound	_	_	_	$\frac{2A^3}{3S \mathbf{f_x} }$	1
Standard lens	$\delta(\mathbf{x} - s_0 \mathbf{u})$	$R( \mathbf{u} /A)$	$\frac{\pi^2 A^4}{16} \operatorname{jinc}^2(\pi A s  \mathbf{f}_{\mathbf{x}} )$	0	0
Narrow aperture	$\delta(\mathbf{x} - s_0 \mathbf{u})$	$R(S\Omega \mathbf{u} )$	$rac{\pi^2}{16S^4\Omega^4} \mathrm{jinc}^2(rac{\pi}{S\Omega}s \mathbf{f_x} )$	$\frac{\pi^2}{16S^4\Omega^4} \operatorname{jinc}^2\left(\frac{\pi}{2\Omega} \mathbf{f_x} \right)$	$\operatorname{jinc}^2(rac{\pi}{2\Omega} \mathbf{f_x} )$
Coded aperture	$\sum_{j} \{\delta(\mathbf{x} - s_0 \mathbf{u})$	$\gamma_j R(rac{u-u_j}{arepsilon A'}) R(rac{v-v_j}{arepsilon A'}) \}$	$\frac{A'^2}{2S^2\Omega^2}\operatorname{sinc}^2\left(\frac{\pi}{S\Omega}sf_x\right)\\\cdot\operatorname{sinc}^2\left(\frac{\pi}{S\Omega}sf_y\right)$	$\frac{A'^2}{2S^2\Omega^2}\operatorname{sinc}^2\left(\frac{\pi}{2\Omega}f_x\right)$ $\cdot\operatorname{sinc}^2\left(\frac{\pi}{2\Omega}f_y\right)$	$\operatorname{sinc}^2\left(\frac{\pi}{2\Omega}f_x\right)\cdot\operatorname{sinc}^2\left(\frac{\pi}{2\Omega}f_y\right)$
Lattice-focal lens	$\sum_{j} \{\delta(\mathbf{x} - s_j \mathbf{u})\}$	$R(\frac{u-u_j}{\varepsilon A'})R(\frac{v-v_j}{\varepsilon A'})\}$	$\frac{\frac{A'^{8/3}}{(S\Omega)^{4/3}} \sum_{j} \operatorname{sinc}^{2} \left( \frac{\pi (s-j\Delta s)}{\Delta s \Omega} f_{x} \right)}{\cdot \operatorname{sinc}^{2} \left( \frac{\pi (s-j\Delta s)}{\Delta s \Omega} f_{y} \right)}$	$ \hat{\phi}_{\Delta s/2}(\mathbf{f_x}) ^2$	$ \hat{\phi}_{\Delta s/2}(\mathbf{f_x}) ^2/ \hat{\phi}_0(\mathbf{f_x}) ^2$
Wavefront coding	$\delta(\mathbf{x} - (au^2, av^2))$	R(u/A')R(v/A')	$\frac{A'^2}{S^2 f_x  f_y }$	$\frac{A'^2}{S^2 f_x  f_y }$	1
Static focus sweep	$rac{1}{S}\int_{-S/2}^{+S/2} \{\delta(\mathbf{x}-s_0\mathbf{u})\}$	$R( \mathbf{u} /A)\}ds_0$	$\frac{A^2}{S^2  \mathbf{f_r} ^2}$	$\frac{A^2}{S^2  \mathbf{f_x} ^2}$	1

	Defocus deblurring	Motion deblurring
High frequency preservation (requires blur estimation)	<b>Coded aperture</b> [Levin et al. 2007; Veeraraghavan et al. 2007] <b>Lattice-focal lens</b> [Levin et al. 2009]	Coded exposure [Raskar et al. 2006] Orthogonal parabolic exposures [Cho et al. 2010] Circular sensor motion [Bando et al. 2011]
Invariant capture (no need for	Wavefront coding [Dowski and Cathey 1995] Focus sweep	Motion-invariant photography (for 1D motion) [Levin et al. 2008]
estimation)	Nagahara et al. 2008] <b>Diffusion coding</b> [Cossairt et al. 2010] <b>Spectral focus sweep</b> [Cossairt and Nayar 2010]	We prove its near depth and 2D motion invariance



### How It Works





		(b)	) Motion deblurring.		
Camera design	3D kernel $k_e(\mathbf{x}, t)$		Squared MTF	High freq. preserv.	MTF invariance
	Integration surface	Integration window	$ \hat{\phi}_{\mathbf{m}}(\mathbf{f}_{\mathbf{x}}) ^2 =  \hat{k}_e(\mathbf{f}_{\mathbf{x}}, -\mathbf{m} \cdot \mathbf{f}_{\mathbf{x}}) ^2$	$\min_{\mathbf{m}}  \hat{\phi}_{\mathbf{m}}(\mathbf{f_x}) ^2$	$\min  \hat{\phi}_{\mathbf{m}} ^2 / \max  \hat{\phi}_{\mathbf{m}} ^2$
Upper bound	_	_	_	$rac{T}{M \mathbf{f_x} }$	1
Static/follow-shot	$\delta({f x}-{f m}_0t)$	R(t/T)	$T^2 \operatorname{sinc}^2(\pi T(\mathbf{m} - \mathbf{m}_0) \cdot \mathbf{f_x})$	0	0
Short exposure	$\delta(\mathbf{x})$	$R(M\Omega t)$	$\frac{1}{M^2\Omega^2}\operatorname{sinc}^2\left(\frac{\pi}{M\Omega}\mathbf{m}\cdot\mathbf{f_x}\right)$	$\frac{1}{M^2\Omega^2}\operatorname{sinc}^2(\frac{\pi}{2\Omega} \mathbf{f_x} )$	$\operatorname{sinc}^2(\frac{\pi}{2\Omega} \mathbf{f_x} )$
Coded exposure	$\sum_{j} \{\delta(\mathbf{x})$	$\gamma_j R(rac{t-t_j}{arepsilon T}) \}$	$\frac{T}{2M\Omega}\operatorname{sinc}^2(\frac{\pi}{M\Omega}\mathbf{m}\cdot\mathbf{f_x})$	$\frac{T}{2M\Omega}\operatorname{sinc}^2(\frac{\pi}{2\Omega} \mathbf{f_x} )$	$\operatorname{sinc}^2(\frac{\pi}{2\Omega} \mathbf{f_x} )$
1D motion-	$\delta(\mathbf{x}-(at^2,0))$	R(t/T)	$rac{T}{M f_{oldsymbol{X}} }R(rac{\mathbf{m}\cdot\mathbf{f_{x}}}{M f_{oldsymbol{X}} })$	$rac{T}{M f_x }$ $(f_y=0)$	$1 (f_y = 0)$
invariant				$0 \ (f_y \neq 0)$	$0 \ (f_y \neq 0)$
Circular motion	$\delta(\mathbf{x}-\mathbf{r}(t))$	R(t/T)	$T^2 J_n^2(rac{MT}{2} \mathbf{f_x} )$	0	0
	with $\mathbf{r}(t) = (r\cos(2t))$	$(\pi t/T), r\sin(2\pi t/T))$	with $n = T\mathbf{m} \cdot \mathbf{f}_{\mathbf{x}}$		
Orthogonal para-	$\delta(\mathbf{x}-(at^2,0))$	R(2t/T) (1st shot)	$\frac{T}{2\sqrt{2}M f_x }R(\frac{\mathbf{m}\cdot\mathbf{f_x}}{\sqrt{2}M f_x })$	$rac{T}{2\sqrt{2}M f_x }R(rac{fy}{2 f_x })$	$1 ( f_y  \le  f_x ), \ 0 (\text{o/w})$
bolic exposures	$\delta(\mathbf{x} - (0, at^2))$	R(2t/T) (2nd shot)	$\frac{T}{2\sqrt{2}M fy }R(\frac{\mathbf{m}\cdot\mathbf{f_{x}}}{\sqrt{2}M fy })$	$\frac{T}{2\sqrt{2}M fy }R(\frac{fx}{2 fy })$	$1 ( f_x  \le  f_y ), 0 (o/w)$
		(c) Joint de	focus and motion deblurring.		
Camera design	5D kernel	$k(\mathbf{x}, \mathbf{u}, t)$	Squared MTF $ \hat{\phi}_{s,\mathbf{m}}(\mathbf{f_x}) ^2$	High freq. preserv.	MTF invariance
	Integration surface	Integration window	$= \hat{k}(\mathbf{f_x},-s\mathbf{f_x},-\mathbf{m}\cdot\mathbf{f_x}) ^2$	$\min_{s,\mathbf{m}}  \hat{\phi}_{s,\mathbf{m}}(\mathbf{f_x}) ^2$	$\min  \hat{\phi}_{s,\mathbf{m}} ^2 / \max  \hat{\phi}_{s,\mathbf{m}} ^2$
Upper bound	_		_	$2A^3T$	1

	integration surface integration window	$=  \kappa(\mathbf{I}_{\mathbf{X}}, -S\mathbf{I}_{\mathbf{X}}, -\mathbf{III} \cdot \mathbf{I}_{\mathbf{X}}) $	$\lim_{s \to \mathbf{m}}  \varphi_s, \mathbf{m}(\mathbf{Ix}) $	
Upper bound		_	$\frac{2A^3T}{3SM \mathbf{f_x} ^2}$	1
Combination of existing designs	$k_a(\mathbf{x}, \mathbf{u}) * k_e(\mathbf{x}, t)$	$ \hat{\phi}_s(\mathbf{f_x}) ^2 \cdot  \hat{\phi}_{\mathbf{m}}(\mathbf{f_x}) ^2$	$\min_s  \hat{\phi}_s ^2 \ \cdot \min_{\mathbf{m}}  \hat{\phi}_{\mathbf{m}} ^2$	$\begin{array}{c} (\min  \hat{\phi}_s ^2 / \max  \hat{\phi}_s ^2) \\ \cdot (\min  \hat{\phi}_{\mathbf{m}} ^2 / \max  \hat{\phi}_{\mathbf{m}} ^2) \end{array}$
Focus sweep	$\delta(\mathbf{x} - wt\mathbf{u}) R( \mathbf{u} /A)R(t/T)$	$\frac{A^3 T}{\sqrt{3}SM \mathbf{f}_{\mathbf{x}} ^2} \left(1 - \frac{4 \mathbf{m} \cdot \mathbf{f}_{\mathbf{x}} ^2}{3M^2 \mathbf{f}_{\mathbf{x}} ^2}\right)$	$\frac{2A^3T}{3\sqrt{3}SM \mathbf{f_x} ^2}$	$\frac{2}{3}$

• 58% of the upper bound in high-frequency preservation

• 67% of the upper bound in depth and 2D motion invariance

# omparison

PSF invariance (center)

PSF invariance (center)

PSF invariance (center)





### Result

